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TABLE OF CONTENTS



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# MATHEMATICAL REVIEWS

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References to reviews in Mathematical Reviews before volume 20 (1959) are by volume and page number, as MR **19**, 532: from volume 20 on, by volume and review number, as MR **20** #4387. Reviews reprinted from Applied Mechanics Reviews, Referativnyĭ Žurnal, or Zentralblatt für Mathematik are identified in parentheses following the reviewer's name by AMR, RŽMat (or RŽMeh, RŽAstr. Geod.), Zbl, respectively.




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# Mathematical Reviews

Vol. 22, No. 3B March, 1961 Reviews 1928–2509

## PROBABILITY

See also 1953.

1928:

**Vincze, István. On some joint distributions and joint limiting distributions in the theory of order statistics. II.** Magyar Tud. Akad. Mat. Kutató Int. Közl. **4** (1959), 29–47. (Hungarian and Russian summaries)

For part I see same Közl. **2** (1957), 3/4, 183–209 [MR **21** #3915]. We use the notations of that review. Let

$$P_{r,k}^{(n)} = P\{\sup_{-\infty<y<+\infty}(F_n(y)-G_n(y)) = k/n,\ F_n(\xi_n+0)+G_n(\xi_n+0) = r/n\}$$

for the case $F=G$. The author determines the generating function of $P_{r,k}^{(n)}$ in a very elementary way and gives some simple applications. He proves, e.g., $P_{2m-1}^{(n)}=P_{2m}^{(n)}$, $1\leq m\leq n$, where $P_r^{(n)}=\sum_k P_{r,k}^{(n)}$. A table containing $P_{r,k}^{(10)}$ for all possible pairs $(r, k)$ is also published.

*L. Schmetterer* (Hamburg)

1929:

**Cohen, A. Clifford, Jr. Estimating the parameters of a modified Poisson distribution.** J. Amer. Statist. Assoc. **55** (1960), 139–143.

The modification consists in changing 1 to 0 with probability $\theta$. With $f_0, f_1, \cdots$ an empirical distribution and $\bar{x}=\sum xf_x$, the maximum likelihood estimation considered requires a maximizer of

$$-\lambda+\bar{x}\log\lambda+f_0\log(1+\theta\lambda)+f_1\log(1-\theta)$$

with respect to $(\lambda, \theta)$ in $]0, \infty[\times[0, 1]$. The author's solution should be corrected by lower truncation of his $\hat{\lambda}$ at $\bar{x}$ and of the resulting $\hat{\theta}$ at 0.

*J. Hannan* (E. Lansing, Mich.)

1930:

**Spitzer, Frank. A Tauberian theorem and its probability interpretation.** Trans. Amer. Math. Soc. **94** (1960), 150–169.

Let $S_k$ be the sum of $k$ independent random variables $X_i$ with common distribution. Let $M_n=\max_{0\leq k\leq n}S_k$. In a previous paper [Trans. Amer. Math. Soc. **82** (1956), 323–339; MR **18**, 156], the author obtained the generating function for the distribution of $M_n$. He has further shown [Duke Math. J. **24** (1957), 327–343; MR **19**, 890] that these probabilistic results yield information concerning certain solutions of the Wiener-Hopf equation (*) $F(x)=\int_0^\infty k(x-y)F(y)dy$ where $k(x)$ is the (known) density of the r.v. $X_i$. In particular he proved the following theorem: Let $k(x)$ be a symmetric probability density function with finite variance $\sigma^2$. Suppose a solution of (*) is defined to mean a nondecreasing function $F(x)$ defined on $0\leq x<\infty$, continuous on the right, with $F(0)>0$. Then (*) has (except for a multiplicative constant) a unique solution $F(x)$. The iterates of (*) defined by $F_0(x)=1$ and $F_{n+1}(x)=\int_0^\infty k(x-y)F_n(y)dy$, $x\geq 0$, converge to a solution $G(x)$ in the following way: $G(x)=\lim_{n\to\infty}(n\pi)^{1/2}F_n(x)$ satisfies (*), is absolutely continuous, nondecreasing, with $G(0)>0$. The solution $G(x)$ has the asymptotic behaviour

$$\lim_{x\to\infty} G(x)/x = 2^{1/2}/\sigma.$$

In this paper it is shown that this theorem remains valid in every respect if (a) the hypothesis that $k$ is symmetric is replaced by the condition that it has mean 0 and (b) the asymptotic behaviour of $G$ is strengthened to $\lim_{x\to\infty}[G(x+h)-G(x)]=h(2^{1/2}/\sigma)$. A probabilistic representation for the generating function of $G$ is given. In the course of the proof the author gives a strengthening of the arc-sine law and an extension of renewal theory.

*J. L. Snell* (Hanover, N.H.)

1931:

**Kawashima, Genkichi. Les générateurs génétiques des matrices stochastiques. II.** Bull. Univ. Osaka Prefecture Ser. A **7** (1959), 185–192.

[For part I see the author, same Bull. **6** (1958), 25–32; MR **21** #1314.]

This paper aims at obtaining a large class of stochastic matrices depending on parameters.

*O. Taussky-Todd* (Pasadena, Calif.)

1932:

**Robinson, Enders A. Sums of stationary random variables.** Proc. Amer. Math. Soc. **11** (1960), 77–79.

The author proves the following theorem for sums of stationary random variables: Let $\{x(t), t\in(-\infty, \infty), t \text{ an integer}\}$ be a second-order stationary process with spectral distribution function $F(u)$. For variance $\{\sum_{-1}^{n} x(t)\}$ to be bounded for all positive integers $n$, each of the following two conditions is necessary and sufficient:

(i) $$\int_{-.5}^{.5} \sin^{-2}\pi u\, dF(u) < \infty;$$

(ii) there is a second-order stationary random process $\{y(t), t\in(-\infty, \infty)\}$ satisfying $y(t)-y(t+1)=x(t)$.

*A. T. Bharucha-Reid* (Eugene, Ore.)

1933:

**Bharucha-Reid, Albert T. Sur les équations intégrales aléatoires de Fredholm à noyaux séparables.** C. R. Acad. Sci. Paris **250** (1960), 657–658.

In this note the notion of a random contraction operator is utilized to generalize some theorems of the theory of Fredholm integral equations to the theory of random Fredholm integral equations.

*W. A. J. Luxemburg* (Pasadena, Calif.)

1934:

**Bharucha-Reid, Albert T. Sur les équations intégrales aléatoires de Fredholm à noyaux séparables.** C. R. Acad. Sci. Paris **250** (1960), 454–456.

In this note the author considers the following random Fredholm integral equation:

$$x(t,\omega)-\lambda\int_{D_\omega}K(t,u)x(u,\omega)dm(u)=y(t),$$

where $x(u,\omega)$ is a random variable with values in an Orlicz space and defined on the product space $U\times\Omega$; here $(\Omega,\alpha,\mu)$ is a probability space and $U$ is a compact subset of $R_k$; $m$ is Lebesgue measure and the kernel $K$ is supposed to be independent of $\omega$ and is a measurable function defined on $D_\omega\times D_\omega$. For the special case that $K(t,u)=\sum_{i=1}^{n}\alpha_i(t)\beta_i(u)$, the author is able to show the following analogue of a classical theorem of the theory of Fredholm integral equations: The problem of solving a random Fredholm integral equation with a kernel of finite rank is equivalent to the problem of solving a finite system of random linear equations.

*W. A. J. Luxemburg* (Pasadena, Calif.)

1935:

**Linnik, Yu. V. General theorems on the factorization of infinitely divisible laws. III. Sufficient conditions (countable bounded Poisson spectrum; unbounded spectrum; "stability").** Teor. Veroyatnost. i Primenen. **4** (1959), 150–171. (Russian. English summary)

This is the third instalment of a comprehensive paper of the author, the first two parts of which appeared in the same journal [**3** (1958), 3–40; **4** (1959), 55–85; MR **20** #4883; **21** #2300]. The decomposition theorem quoted in the first review is extended to the case of unbounded Poisson spectra as follows (we use the notations of the quoted review). If the characteristic function of an infinitely divisible law $F$ satisfies the necessary conditions enumerated in the review and if, furthermore, we have $\log\log 1/\lambda_m>c\mu_m^{1+\alpha}$, $\log\log 1/\lambda_{-n}>c\nu_n^{1-\alpha}$ for some $\alpha>0$, $c>0$ and all sufficiently large $\mu_m$, $\nu_n$ then $F$ has only infinitely divisible components, i.e., the necessary conditions are, under the stated further assumptions, sufficient as well. This clearly extends the results of the first part concerning the case of bounded Poisson spectrum.

*A. Dvoretzky* (Jerusalem)

1936:

**Linnik, Yu. V. On "$\alpha$-factorizations" of infinitely divisible probability laws.** Vestnik Leningrad. Univ. **14** (1959), no. 1, 14–23. (Russian. English summary)

Continuing earlier investigations [Dokl. Akad. Nauk SSSR **116** (1957), 735–737; MR **20** #331; and the paper reviewed above, as well as its two predecessors] the author proves the following theorem: Let $t_k$ be a null-sequence of real numbers and assume that we have

$$(1)\qquad f_1(t_k)^{\alpha_1}\cdots(f_s(t_k))^{\alpha_s}=\varphi(t_k)$$

where $\alpha_j>0$, $f_j$ are characteristic functions of random variables and $\varphi$ is the characteristic function of an infinitely divisible law whose logarithm has the form referred to in the preceding review with $\gamma\geq 0$ [note that the Gaussian component may vanish]. Then (1) is valid for all real and complex $t_k$ and the logarithms of $f_j$ have the same form as that assumed above about the logarithm of $\varphi$. The proof of the theorem is achieved through reducing it to results in the paper reviewed above and its predecessors.

*A. Dvoretzky* (Jerusalem)

1937:

**Rényi, Alfréd. On the probabilistic generalization of the large sieve of Linnik.** Magyar Tud. Akad. Mat. Kutató Int. Közl. **3** (1958), 199–206. (Hungarian and Russian summaries)

Let $\eta$ be a random variable with finite variance and let $\{\xi_n\}$ be a sequence of discrete random variables. Denote by $z_{nk}$ $(k=1,2,\cdots)$ the possible values of $\xi_n$ and write $A_{nk}=\{\xi_n=z_{nk}\}$. Let $\varphi(\xi_n,\xi_m)$ be the mean square contingency of $\xi_n$ and $\xi_m$, i.e.,

$$\varphi^2(\xi_n,\xi_m)=\sum_k\sum_l\left\{\frac{[P(A_{nk}A_{ml})-P(A_{nk})P(A_{ml})]^2}{P(A_{nk})P(A_{ml})}\right\}$$

and $\theta_{\xi_n}(\eta)$, the correlation ratio of $\eta$ on $\xi_n$, i.e.,

$$\theta_{\xi_n}^2(\eta)=\frac{D^2\{E\{\eta|\xi_n\}\}}{D^2\{\eta\}}.$$

The author, generalizing his earlier result [Ann. Inst. Fourier Grenoble **1** (1949), 43–52; MR **14**, 886], proves that if

$$\left|\sum_{n\neq m}\sum\varphi(\xi_n,\xi_m)x_nx_m\right|\leq B\sum_n x_n^2$$

provided that $\sum_n x_n^2<\infty$, then

$$\sum_{n=1}^{\infty}\theta_{\xi_n}^2(\eta)\leq(1+B).$$

*L. Takács* (New York)

1938:

**Rényi, A. New version of the probabilistic generalization of the large sieve.** Acta Math. Acad. Sci. Hungar. **10** (1959), 217–226. (Russian summary, unbound insert)

Let $\eta$ and $\xi$ be two random variables. Denote by $R(\xi,\eta)$ their correlation coefficient, and by $\theta_\xi(\eta)$ the correlation ratio of $\eta$ on $\xi$. Following H. Gebelein [Z. Angew. Math. Mech. **21** (1941), 364–379; MR **4**, 104] the author defines the maximal correlation $S(\xi,\eta)$ as follows:

$$S(\xi,\eta)=\sup_{f,g}R(f(\xi),g(\eta))$$

where $f$ and $g$ run through all Borel measurable functions for which $R(f(\xi),g(\eta))$ is defined.

The author proves the following new version of the large sieve. Let $\xi_n$, $1\leq n<\infty$, be an infinite sequence of random variables for which there exists a constant $C$ so that

$$\left|\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}S(\xi_n,\xi_m)x_nx_m\right|<C\sum_{n=1}^{\infty}x_n^2.$$

Let $\eta$ be an arbitrary random variable for which $M(\eta^2)$ exists. Then we have

$$\sum_{n=1}^{\infty}\theta_{\xi_n}^2(\eta)\leq C.$$

Loosely stated the theorem means that if we have a sequence of nearly independent variables $\xi_n$, then an arbitrary random variable $\eta$ can not depend too strongly on too many of them. This result contains all previous versions of the large sieve. [See A. Rényi, J. Math. Pures

Appl. (9) **28** (1949), 137–149; Compositio Math. **8** (1950), 68–75; MR **11**, 161, 581; and #1937 above.]

The author also proves some theorems about various measures of dependence of random variables.

*P. Erdös* (Budapest)

1939:

**Krishna Iyer, P. V.; Ray, D. Low order moments of some distributions arising from two-state Markoff chains.** J. Indian Soc. Agric. Statist. **10** (1958), 23–32, xvi. (Hindi summary)

Consider a two-state Markoff chain with states $A$ and $B$. The author computes the first two moments for the distribution of $AA$, $AB$ and $AB$ and $BA$ joins in the first $n$ trials. The covariance for the number of $AA$ and $AB$ joins is also given. *J. L. Snell* (Hanover, N.H.)

1940:

**Cheng, Shaw-lian. On the regular and singular stationary generalized stochastic processes.** Sci. Record (N.S.) **3** (1959), 352–358.

L'A. utilise les définitions de K. Urbanik. Soit $\Phi(\omega, t)$ un processus stochastique généralisé stationnaire. Ce processus est appelé singulier si pour tout $\tau > 0$, il existe deux suites de nombres $\{\alpha_j\}$ et $\{t_j\}$, telles que $\sum_{j=1}^{\infty} \alpha_j \Phi(\omega, t+t_j) \Rightarrow \Phi(\omega, t+\tau)$. Il démontre que cette définition équivaut à l'égalité $\int_{-\infty}^{\infty} |\log H'(\lambda)|/(1+\lambda^2)\, d\lambda = +\infty$, où $dH$ est la mesure spectrale qui correspond à la corrélation $B$ de $\Phi$ ($B(t) = \int_{-\infty}^{\infty} e^{it\lambda} dH(\lambda)$). Ensuite, le processus $\Phi$ est dit régulier si $\int_{-\infty}^{\infty} |\log H'(\lambda)|/(1+\lambda^2)\, d\lambda < +\infty$. On démontre que tout processus généralisé stationnaire est la somme orthogonale d'un processus régulier et d'un processus singulier. *G. Marinescu* (Bucharest)

1941:

**Bellman, Richard; Kalaba, Robert. Invariant imbedding, random walk and scattering. II. Discrete versions.** J. Math. Mech. **9** (1960), 411–419.

From the authors' summary: "In the first paper of this series [Proc. Nat. Acad. Sci. U.S.A. **43** (1957), 930–933; MR **20** #6158], we sketched the application of the techniques of invariant imbedding to various random walk processes and questions of scattering theory. In this paper, we wish to fill in the details and to show how the same ideas enable us to treat multi-dimensional, time-dependent, and energy-dependent processes."

*J. Wolfowitz* (Ithaca, N.Y.)

1942:

**Haight, Frank A.; Breuer, Melvin Allen. The Borel-Tanner distribution.** Biometrika **47** (1960), 143–150.

The Borel-Tanner distribution is defined for $\alpha > 0$ and for $r$ a positive integer by

$$p(x; r, \alpha) = A(x, r) e^{-\alpha x} \alpha^{x-r} \quad (x = r, r+1, \cdots),$$

where $A(x, r) = r x^{x-r-1}/(x-r)!$. In queuing theory $p(x; r, \alpha)$ represents the probability that exactly $x$ members of a queue will be served before the queue first vanishes, assuming there were $r$ members to start and $\alpha = \lambda/\mu$ is the traffic intensity with Poisson arrivals and constant service time. The authors examine this distribution for $\alpha < 1$ and obtain the mean, $r/(1-\alpha)$, and the variance, $r\alpha/(1-\alpha)^3$. They also obtain several different generating functions from which higher moments can be obtained. Several other properties of the distribution are developed and tables are given for the values $r = 1$, $\alpha = 0.01,\ 0.02, \cdots, 0.62$ for the function in cumulative form, $P(x; r, \alpha) = \sum_{y=r}^{x} p(y; r, \alpha)$ up to the first value of $P > 0.999$. *H. M. Gurk* (Princeton, N.J.)

1943:

**Boyer, R. H. An integro-differential equation for a Markov process.** J. Soc. Indust. Appl. Math. **7** (1959), 473–486.

The author considers the following stochastic process $\{X(t);\ 0 \leq t < \infty\}$ which arises in the theory of inventories: $X(t)$ assumes only nonnegative values. Starting from $t = 0$ with the value $X(0)$, $X(t)$ increases linearly and at the instants $t_n$ $(n = 1, 2, \cdots)$ there is a jump of a certain (positive or negative) magnitude $X_n$. If at time $t$ a negative jump occurs and is large enough to pass the zero line then $X(t)$ is to be taken as 0. The sequence $\{t_n\}$ forms a Poisson process and $\{X_n\}$ are identically distributed independent random variables which are independent of $\{t_n\}$ too.

$$P\{X_n \leq x \mid X_n > 0\} = 1 - \sum_{j}^{N} a_j e^{-\nu_j x} \quad \text{for } x > 0,$$

$$P\{-X_n \leq x \mid X_n < 0\} = 1 - \sum_{j=1}^{N'} a'_j e^{-\nu'_j x} \quad \text{for } x > 0.$$

The author determines $\varphi(s, u) = \int_0^{\infty} e^{-ut} E\{e^{-sX(t)}\} dt$, $\Phi(x, u) = \int_0^{\infty} e^{-ut} P\{X(t) \leq x\} dt$, the steady state distribution, the expectation of $X(t)$ and deals with the problem of the first passage to zero. *L. Takács* (New York)

1944:

**Lubbock, J. K. On a class of semi-separable processes.** J. Electronics Control (1) **8** (1960), 67–79.

Author's summary: "A class of semi-separable processes can be defined for which the $g$ function separates over the range $t_1 \geq t_2$ thus:

$$g(x_1; t_1, t_2) = \int [x_2 - \langle x_2(t_1) \rangle] p_2(x_1, x_2; t_1, t_2) dx_2$$

$$= g_1(x_1, t_1) g_2(t_1, t_2)$$

for $t_1 \geq t_2$ only. This class of processes possesses all the properties of Nuttall's separable class, including the invariance property, for $t_1 \geq t_2$ only. It is shown that semi-separability and separability are preserved under linear transformation of $x_2$. An example of this class of processes is given and its importance in non-linear filtering and prediction is mentioned."

1945:

**Samuels, J. Clifton. On the stability of random systems and the stabilization of deterministic systems with random noise.** J. Acoust. Soc. Amer. **32** (1960), 594–601.

Author's summary: "A general theory of mean square stability of random linear systems is developed when several system parameters vary as white noise stochastic processes. It is found that stability in mean square is determined from the character of the roots of a determinantal equation involving the Fourier transforms of

double products of the weighting functions of the 'average' system and the spectral densities of the parameter processes.

"The general theory is applied to the mean square stability of an RLC circuit in which the resistance and capacitance have purely random fluctuations.

"In the course of the study, a new type of dynamic stability is predicted, namely, the possibility of stabilizing unstable deterministic systems with random noise. Preliminary experimental studies appear to confirm this theoretical prediction." *R. A. Silverman* (New York)

## STATISTICS

See also 2004, 2454, 2498, 2499.

1946:

**Austin, Thomas L., Jr. An approximation of the point of minimum aggregate distance.** Metron **19**, no. 3–4, 10–21 (1959).

The author gives a practical method for the construction of the points of minimum aggregate distance or median—after the term used in the paper—that realizes the minimum of $\sum_j \overline{MA}_j$, where $\overline{MA}_j$ are the distances of $M$ to the $n$ points $A_j$ given in the plane. One proceeds in the following manner. Select a point $m_1$ at random on the plane and draw a circle $C$ having the center $m_1$ and the radius $r_1$. Let $M_1$ be the center of the points of intersection of the lines $m_1A_j$ with the circle. Select then a new trial point $m_2$ about half way between $m_1$ and $M_1$ and repeat the process. The sequence $m_1, m_2, \cdots$ will very soon converge—following the author—to the median $m$.

*O. Onicescu* (Bucharest)

1947:

**Georgescu-Roegen, Nicholas. On the extrema of some statistical coefficients.** Metron **19**, no. 3–4, 38–45 (1959).

The paper deals with proving four theorems concerning the system of certain statistical indices, two of them Benedetti's, obtaining a new demonstration, the other two being completely new theorems. The latter concern the extrema of the sums $S_k = \sum_i x_i^k$ with $\sum x_i = 0$, $\sum_i x_i^2 = 1$, namely, the values of $\max|S_{2j+1}|$, $\max S_{2j}$, $\min S_{2j}$. Hence follow well-defined expressions for the extremal values of the coefficients of Poisson of even or of odd order. *C. Calinescu* (Bucharest)

1948:

**Barton, D. E.; David, F. N.; Fix, Evelyn. The polykays of the natural numbers.** Biometrika **47** (1960), 53–59.

This is a study of calculations of moments and of certain functions of moments which arise in sampling without replacement from a finite population. There are applications to moments of rank order statistics.

*M. Dwass* (Evanston, Ill.)

1949:

**Burr, E. J. The distribution of Kendall's score $S$ for a pair of tied rankings.** Biometrika **47** (1960), 151–171.

Details and tables of the null distribution are given, particularly for samples not larger than 6.

*I. R. Savage* (Cambridge, Mass.)

1950:

**Laha, R. G.; Lukacs, E. On certain functions of normal variates which are uncorrelated of a higher order.** Biometrika **47** (1960), 175–176.

1951:

**Watanabe, S. Correlation indices.** Nuovo Cimento (10) **13** (1959), supplemento, 576–582.

Consider $n$ stochastic variables $y_1, y_2, \cdots, y_n$, each of which can take $g$ discrete values or states $v_1, v_2, \cdots, v_g$. The author is interested in finding a measure of correlation which satisfies the following conditions. (1) The measure is independent of the values $v_t$ assigned to the states. (2) The measure is always non-negative. (3) The total correlation $C^{(n)}$ is decomposed as $C^{(n)} = \sum_{r=1}^{n} T^{(r)}$, where $T^{(r)}$ is non-negative and measures the strength of the correlation peculiar to a subset of $r$ variables taken out of the $n$ variables. He limits himself to two cases: (1) the case of a symmetric set of stochastic variables (the basic distributions are unchanged by a permutation of the indices) and (2) temporally stationary variables. The "information" function of $r$ variables is defined by

$$S^{(r)} = -\sum_{x_1}^{g} \cdots \sum_{x_r}^{g} p^{(r)}(x_1, \cdots, x_r) \log p^{(r)}(x_1, \cdots, x_r).$$

The author chooses as correlation measure the quantity $C^{(n)} = nS^{(1)} - S^{(n)}$. It is verified that this quantity has the desired properties. *J. L. Snell* (Hanover, N.H.)

1952:

**Kuiper, Nicolaas H. Tests concerning random points on a circle.** Nederl. Akad. Wetensch. Proc. Ser. A **63** = Indag. Math. **22** (1960), 38–47.

The author remarks that if $X_1, X_2, \cdots$ are independent random variables distributed over (0, 1) and $F_n(x, a)$ is the empirical distribution function of the sequence $\{X_i + a\} \bmod 1$, $i = 1, 2, \cdots, n$, then the usual Kolmogorov statistic is not independent of $a$. However, he notices that the statistic

$$\sup_x n^{1/2}(F_n(x, a) - x) - \inf_x n^{1/2}(F_n(x, a) - x)$$

is independent of $a$, and suggests this as a test statistic of the hypothesis that the $X_i$ are uniformly distributed over (0, 1). The asymptotic distribution of the statistic to terms of order $1/n$ is given, briefly tabulated, and compared with empirical values. A two-sample analogue is treated and applications to testing the isotropy of flight directions of migratory birds are discussed.

*D. A. Darling* (Ann Arbor, Mich.)

1953:

**Rider, Paul R. Variance of the median of samples from a Cauchy distribution.** J. Amer. Statist. Assoc. **55** (1960), 322–323.

Exact values of the variance of the medians of small samples from the Cauchy distribution are given. Comparison with corresponding values based on the formula for asymptotic variance shows that for small samples the asymptotic formula considerably underestimates the true variance. *G. E. Noether* (Boston, Mass.)

1954:

**Berkson, Joseph. Nomograms for fitting the logistic function by maximum likelihood.** Biometrika **47** (1960), 121–141.

In the logistic model for quantal response to a graded dose, the probability of response at dose-level $x_i$ is $[1+\exp(-\alpha-\beta x_i)]^{-1}$. If the absolute frequency of response at $x_i$ is $r_i$ out of $n_i$ trials, $i=1, \cdots, k$, the maximum-likelihood estimates $\hat{\gamma}$ and $\hat{\beta}$ of $-\alpha/\beta$ and $\beta$, respectively, depend on $\sum r_i$ and $\sum r_i x_i$. For practical ranges of $\sum r_i$ and $\sum r_i x_i$, these nomograms give $\hat{\gamma}$ and $\hat{\beta}$ to two significant figures (with the third usually estimable) for the case of equally spaced $x_i$, $n_i=n$ and $k=3, 4, 5, 6$. Tables are given of the standard errors for large $n$ of $n^{1/2}\hat{\gamma}$ and $n^{1/2}\hat{\beta}$. *M. Stone* (Princeton, N.J.)

1955:

**Weichselberger, K. Über die Parameterschätzung bei Kontingenztafeln, deren Randsummen vorgegeben sind. I.** Metrika **2** (1959), 100–130.

In einer Kontingenztafel bezeichne $p_{ij}$ die Wahrscheinlichkeit, daß ein Element der $i$-ten Zeile und $j$-ten Spalte der Tafel angehöre. Es sei $\sum_i p_{ij}=p_{.j}$, $\sum_j p_{ij}=p_{i.}$. Für alle $i$ und $j$ seien $p_{i.}$ und $p_{.j}$ bekannt und unter dieser Nebenbedingung wird die Lösbarkeit der Maximum-Likelihood Gleichungen und die Konsistenz der Lösungen für die Schätzung der $p_{ij}$ untersucht. Der Autor modifiziert zu diesem Zweck den vom Referenten [*Einführung in die mathematische Statistik*, Springer-Verlag, Vienna, 1956; MR **18**, 681] gegebenen Beweis für die Lösbarkeit der Maximum-Likelihood Gleichungen. Es muß jedoch darauf hingewiesen werden, daß die Untersuchungen des Autors in engster Beziehung zu einem Satz von H. Cramér stehen [*Mathematical models of statistics*, Princeton Univ. Press, Princeton, N.J., 1946; MR **8**, 39; cf. p. 426]. *L. Schmetterer* (Hamburg)

1956:

**Pawlik, K. Der maximale Kontingenzkoeffizient im Falle nichtquadratischer Kontingenztafeln.** Metrika **2** (1959), 150–166. (English summary)

Consider an $r\times s$ contingency table and let $m=\min(r, s)$. It is proved that $\sqrt{((m-1)/m)}$ is an upper bound for Pearson's coefficient of contingency which is obtained when and only when each row ($r\geqq s$) or each column ($r\leqq s$) of the table contains exactly one frequency $\neq 0$. But this is very well known [cf. e.g., H. Cramér, *Mathematical models of statistics*, Princeton Univ. Press, Princeton, N.J., 1946; MR **8**, 39; p. 443]. The relation between Pearson's coefficient of contingency and Gebelein's coefficient of maximal correlation is also discussed. *L. Schmetterer* (Hamburg)

1957:

**Huron, Roger. Note sur un problème d'estimation de paramètres; application aux groupes sanguins.** C. R. Acad. Sci. Paris **250** (1960), 1603–1604.

1958:

**John, S. The distribution of Wald's classification statistic when the dispersion matrix is known.** Sankhyā **21** (1959), 371–376.

On individuals, measurements $x_1, x_2, \cdots, x_p$ with respect to $p$ characteristics are taken. The individuals belong to one of two populations $P_1$ and $P_2$ in which $x_1, x_2, \cdots, x_p$ have a multivariate normal distribution for which the means differ between $P_1$ and $P_2$ but the variance-covariance matrix is the same. The classification problem is to correctly assign a particular person whose measurements have the values, $y_1, y_2, \cdots, y_p$ to $P_1$ or $P_2$. Fisher [Ann. Eugen. **1936**, 179–188] gave a criterion in the case that the two sets of means and the common variance-covariance matrix are known, and Wald [Ann. Math. Statist. **15** (1944), 145–162; MR **6**, 9] gave one analogous in form if the means and the variance-covariance matrix are estimated from samples of $n_1$ and $n_2$ from $P_1$ and $P_2$ respectively, but he did not give an explicit expression for its distribution. In the case the variance-covariance matrix, $\Sigma$, is known but the means are not the author derives the distribution of the criterion statistic, $V=(\bar{x}_1-\bar{x}_2)\,\Sigma^{-1}y'$, in which $\bar{x}_1, \bar{x}_2$ are the two vectors of sample means and $y=(y_1, y_2, \cdots, y_p)$. $V$ is immediately reducible to the form $T=u_1v_1+u_2v_2+\cdots+u_pv_p$ in which $u_1, v_1, u_2, v_2, \cdots, u_p, v_p$ are independent normal variables with unit variances. Accordingly as $p$ is even or odd the sought distribution is found as one of two series, each term of which is expressible as a series. *C. C. Craig* (Ann Arbor, Mich.)

1959:

**Basmann, R. L. On the asymptotic distribution of generalized linear estimators.** Econometrica **28** (1960), 97–107.

This article deals with the proof of asymptotic normality of linear estimators of regression coefficients in systems of equations. The article should be read in connection with the author's paper in Econometrica **25** (1957), 77–83 [MR **19**, 74]. *T. Haavelmo* (Oslo)

1960:

**Watson, G. S. More significance tests on the sphere.** Biometrika **47** (1960), 87–91.

Using ratios of maximised likelihoods, the author derives a test of the hypothesis that the mean vectors of each of several sets of magnetisation vectors obeying Fisher's distribution are coplanar. *P. Whittle* (Cambridge, England)

1961:

**Goodman, Leo A. Partial tests for partial taus.** Biometrika **46** (1959), 425–432.

The author draws attention to plausible situations in which the use of partial tau, as defined by M. G. Kendall, as a coefficient of partial rank correlation may be misleading. By neglecting some of the information contained in the ranking it is possible to construct a series of coefficients of partial rank correlation which are more appropriate and for which there exist relatively simple tests of significance. *P. A. P. Moran* (Oxford)

1962:

**Clunies-Ross, C. W.; Riffenburgh, R. H. Geometry and linear discrimination.** Biometrika **47** (1960), 185–189.

The discrimination problem considered is that of classifying a sample into one of two known multivariate normal populations. From the class of linear statistics,

the discriminator which minimizes the maximum (possibly weighted) conditional probabilities is characterized in terms of certain tangent hyperplanes.

*I. Olkin* (Minneapolis, Minn.)

1963:

**Johnson, N. L. An approximation to the multinomial distribution: some properties and applications.** Biometrika **47** (1960), 93–102.

The multinomial distribution of the frequencies, $f_1, \cdots, f_k$, out of a sample of size $N$, of events having probabilities $\pi_1, \cdots, \pi_k$ ($\sum_{i=1}^{k} \pi_i = 1$), may be approximated by the joint distribution of $Y_1, \cdots, Y_k$, where $Y_j = V_j / \sum_{i=1}^{k} V_j$, where $V_1, \cdots, V_k$ have independent $\chi^2$ distributions with respective degrees of freedom $\nu_1, \cdots, \nu_k$, and where $\nu_j = 2(N-1)\pi_j$. It is shown that this approximation gives the correct means, variances, and covariances. Certain other consequences of this approximation are investigated. A table is provided showing how well the approximation works for the binomial ($k=2$), for values of $m = 25$, 50, and values of $\pi_1 = .1, .3, .5$.

*T. S. Ferguson* (Los Angeles, Calif.)

1964:

**Montello, Jessé. On extension of the concept of asymptotically normal distribution to multidimensional random variables.** Estadística **17** (1959), 457–475. (Portuguese. English summary)

Author's summary: "The author intends to give a new definition of an asymptotically normal multidimensional random variable generalizing the definition of a unidimensional variable. The importance of the definition is that it permits, for properties of asymptotically normal multidimensional random variables, the maintenance of demonstrations analogous and statements equal to those pertaining to the unidimensional case."

1965:

**Singh, B. N. On the applications of the statistics $W$'s and $T$'s for testing two samples.** J. Indian Soc. Agric. Statist. **10** (1958), 107–130.

For several of the nonparametric procedures introduced by Krishna Iyer and Singh [same J. **7** (1955), 127–168; MR **19**, 188] the author gives examples of applications, Monte Carlo calculations, exact null distributions (for small sample sizes), and means and variances (for large sample sizes). *I. R. Savage* (Cambridge, Mass.)

1966:

**Wilks, S. S. Non-parametric statistical inference.** Probability and statistics: The Harald Cramér volume (edited by Ulf Grenander), pp. 331–354. Almqvist & Wiksell, Stockholm; John Wiley & Sons, New York; 1959. 434 pp. $12.50.

The topics presented are outlined by the author as follows. "Within the past twenty years many results have been obtained on non-parametric inference. But these results have been spotty and piecemeal. What might be regarded as good non-parametric solutions have been found for the problem of tolerance limits and tolerance regions, confidence intervals for quantiles, and confidence bands for a continuous cdf $F(x)$. On the other hand, there has not yet emerged a general theory of non-parametric hypothesis testing as complete and comprehensive as that which has been developed for parametric hypothesis testing." Following this list of topics the paper presents, without claiming completeness, a lucid and systematic survey of non-parametric statistical theory, covering the "good solutions" as well as the less satisfactory attempts at a general theory of non-parametric hypothesis testing.

*Z. W. Birnbaum* (Seattle, Wash.)

1967:

**Seal, K. C. On ranking parameters of scale in Type III populations.** J. Amer. Statist. Assoc. **53** (1958), 164–175.

Consider $n+1$ populations with Gamma probability densities $(a_i^k/\Gamma(k))e^{-a_i x}x^{k-1}$, $a_i > 0$, $k \geq 1$, $i = 0, 1, \cdots, n$, where $k$ is known, but the scale parameters $a_i^{-1}$ are not known. When $a_i > a_j$, the $i$th population is called "more desirable" than the $j$th. The author presents a decision rule which solves the following problem. For given $\alpha$, $0 < \alpha < 1$, one wishes to select from the $n+1$ populations a number $\nu(\alpha)$, not known in advance, so that several reasonable requirements are fulfilled, of which we reproduce here only the first three: (A) the probability of including the most desirable population, i.e., the one with the largest $a_i$, in the $\nu(\alpha)$ population selected by the rule is maximum among all $\binom{n+1}{\nu(\alpha)}$ possible groups of size $\nu(\alpha)$; (B) the probability of not including the most desirable population in the selected group is $\leq \alpha$; (C) the probability of not including the most desirable population is $\leq$ the probability of not including any other population.

*Z. W. Birnbaum* (Seattle, Wash.)

1968:

**Saw, J. G. A note on the error after a number of terms of the David-Johnson series for the expected values of normal order statistics.** Biometrika **47** (1960), 79–86.

David and Johnson [Biometrika **41** (1954), 228–240; MR **16**, 382] developed an infinite series for the expected value of the $r$th smallest of $n$-ordered normal variates in which the terms are the products of moments of the Beta distribution and the derivatives of the inverse normal integral. In this paper the author develops a bound for the error when only a finite number of terms of the series are used, and then compares the bound with that given by Plackett [Ann. Math. Statist. **29** (1958), 131–142; MR **20** #375] for a different infinite series. The author concludes that Plackett's series seems to converge a little more rapidly, but that computational advantages favor the David and Johnson series.

*D. Teichroew* (Stanford, Calif.)

1969:

**Guttman, Louis. Metricizing rank-ordered or unordered data for a linear factor analysis.** Sankhyā **21** (1959), 257–268.

For a population $P$ of subjects and a set $J$ of tests, let $s_{jp}$ be the score of subject $p$ on test $j$, which may be a rank order or an ordered category for subjects on test $j$ as well as an actual score. The problem considered is that of assigning new scores, $x_{jp}$, which preserve rank-orders within tests, and which have linear regressions on each other. Even considering only bivariate regressions, it is shown that at most one such set of $x_{jp}$'s exists. The consequences of this for linear factor analysis and related matters are discussed. *C. C. Craig* (Ann Arbor, Mich.)

1970:

**Elfving, G. Design of linear experiments.** Probability and statistics: The Harald Cramér volume (edited by Ulf Grenander), pp. 58–74. Almqvist & Wiksell, Stockholm; John Wiley & Sons, New York; 1959. 434 pp. $12.50.

This paper is a survey of work in design of experiments which has recently attracted considerable attention from statisticians. This work centers about the design of optimal experiments for the estimation of parameters. The author relates his own pioneering work on geometric allocation theory to results of Hotelling on weighing, and Moriguti and Ehrenfeld on orthogonality and minimax designs, and gives unpublished work by Savage on admissibility of experiments. The paper ends with the remark that one unsolved problem is that of finding numerical procedures, and with a note added in proof referring to a recent article by Kiefer and Wolfowitz relating optimum allocation and game theory. Almost all results here apply to linear experiments, i.e., experiments in which the expected values of the observed variables are linear functions of the unknown parameters. Some of the results do extend, as the author remarks, to more general experiments. *H. Chernoff* (Stanford, Calif.)

1971:

**Quenouille, M. H. Experiments with mixtures.** J. Roy. Statist. Soc. Ser. B **21** (1959), 201–202.

Author's summary: "The difference is discussed between a model for experiments with mixtures proposed recently by Scheffé [J. Roy. Statist. Soc. Ser. B. **20** (1958), 344–360; MR **20** #6766] and an earlier model of the author's."

1972:

**Špaček, A. Processus aléatoires de décision statistique conditionnée.** Le calcul des probabilités et ses applications. Paris, 15–20 juillet 1958, pp. 157–164. Colloques Internationaux du Centre National de la Recherche Scientifique, LXXXVII. Centre National de la Recherche Scientifique, Paris, 1959. 196 pp.

Modifying somewhat the Wald set-up the author studies decision problems where the parameter space and the decision space consist of two points each. Were the a priori probabilities known the statistician would employ Bayes solutions; in the present situation this a priori information is not available and the statistician proceeds by estimating the parameters as best he can in the light of past experience. The author establishes some convergence results for such schemes.

*A. Dvoretzky* (Jerusalem)

1973:

**Bhate, D. H. Approximation to the distribution of the sample size for sequential tests. II. Tests of composite hypotheses.** Biometrika **47** (1960), 190–193.

Methods given by the author to approximate the average sample size for a sequential test of a simple hypothesis are here extended to some tests of composite hypotheses. In particular the problem of testing the hypothesis $\lambda=\lambda_0$ against $\lambda=\lambda_1$ when $X_{ij}$ are NID$(\mu_i, \sigma_i^2)$ $(i=1, 2; j=1, 2, 3, \cdots)$ and $\lambda=\sigma_1^2/\sigma_2^2$ is studied.

*D. G. Chapman* (Seattle, Wash.)

1974:

**Vagholkar, M. K.; Wetherill, G. B. The most economical binomial sequential probability ratio test.** Biometrika **47** (1960), 103–109.

A batch of $n$ items is presented for acceptance inspection in which the items must be classified as either defective or non-defective. Let $p$ denote the fraction defective and assume $p$ has the prior distribution $\Pr(p=p_1)=a_1$ and $\Pr(p=p_2)=1-a_1$, where $p_1<p_0<p_2$, $p_0$ is a critical fraction defective, i.e., batches with $p<p_0$ should be accepted and $p>p_0$ should not be accepted.

Further let $W_{21}$ [$W_{12}$] denote the cost if a batch with fraction defective $p_1$ [$p_2$] is rejected [accepted] and let $c$ denote the cost of inspecting a single item. The most economical sampling plan within this framework is determined. The results are expressed in terms of the score notation for sequential sampling due to Barnard [Suppl. J. Roy. Statist. Soc. 8 (1946), 1–21; MR 8, 395]. Some extensions are also considered.

*D. G. Chapman* (Seattle, Wash.)

1975:

**Ishii, Gorô. On a non-parametric test in life test.** Bull. Math. Statist. **8**, 73–79 (1959).

Suppose $X_1, \cdots, X_M$ are $M$ independent and identically distributed random variables having a continuous distribution function $G(x)$. Let $F(x)$ be a continuous distribution function, and let $a_1, \cdots, a_{m-1}$ be determined by $F(a_i)=i/m$ $(i=1, \cdots, m-1)$. Let $V$=number of intervals among $(-\infty, a_1]$, $(a_1, a_2]$, $\cdots$, $(a_{n-1}, a_n]$ which contain none of $X_1, \cdots, X_M$. Consider the hypothesis that $G(x)\equiv F(x)$. A test suggested for testing this hypothesis is: Reject hypothesis if $V$ is too large. The actual and asymptotic distributions of $V$ are determined under null and alternative hypotheses, and the power function of the test is studied. The results are placed in a life testing context. *M. Dwass* (Evanston, Ill.)

1976:

**Guthrie, D., Jr.; Johns, M. V., Jr. Bayes acceptance sampling procedures for large lots.** Ann. Math. Statist. **30** (1959), 896–925.

The authors consider the problem of fixed size acceptance sampling procedures from the Bayes point of view. The authors give two reasons for this: First that in most practical situations the statistician will have some subjective a priori distribution, and second that the Bayes procedures form a complete admissible class. In the reviewer's opinion the second argument is inadequate, as it does not preclude the a priori distribution used from varying with the lot size; and from the standpoint of statistical decision theory [see e.g., Savage, *The foundations of statistics*, Wiley, New York, 1954; MR **16**, 147], the first argument is always sufficient.

The specific problem considered is that of inspecting $n$ items out of a lot of size $N$, replacing the defectives among the inspected items, and accepting or rejecting the remainder. It is assumed that the distribution belongs to the exponential family with unknown parameter; only certain families are discussed in detail. Explicit asymptotic formulas are obtained for the Bayes risk and Bayes sample size, and for certain cases it is shown that, if the lot size $N$ is sufficiently large, so that 100% inspection is not optimal, the optimal sample size $n$ is comparable to $\log N$. *H. Rubin* (E. Lansing, Mich.)

1977:

**Ungar, Peter. The cutoffpoint for group testing.** Comm. Pure Appl. Math. **13** (1960), 49–54.

Group testing is a procedure for reducing the expected number of tests required to detect all the members of a population that have a given characteristic. If a single test applied to a group of $m$ individuals can determine whether at least one of them has the characteristic in question, group testing may reduce the expected number of tests required since, if the test shows the group to be free of the characteristic, a single group test does the work of $m$ individual tests. More precisely, the author proves: There exists a group test plan for which the expected number of tests is less than the number of individuals in the population if the probability that any particular individual has the characteristic is less than $\frac{1}{2}(3-\sqrt{5})$, and not otherwise. The major step in the proof is to represent a test plan as a tree and to show that if the hypothesis of the theorem is violated and if any group tested has more than one individual then the expected number of tests can be reduced by testing one of the members of that group individually.

*R. Dorfman* (Cambridge, Mass.)

1978:

**Grenander, Ulf.; Pollak, H. O.; Slepian, D. The distribution of quadratic forms in normal variates: a small sample theory with applications to spectral analysis.** J. Soc. Indust. Appl. Math. **7** (1959), 374–401.

The authors are concerned with the problem of getting reasonable approximations and computing techniques to determine the probability distribution of a quadratic form in normal variables. Let $Q=\sum_{i,j=1}^{n} w_{ij}x_ix_j$ where $W=(w_{ij})$ is a nonnegative definite symmetric matrix and the $x_i$ are jointly normal with mean zero and covariance matrix $R$. Let the $\lambda_i$, $i=1, \cdots, n$, be the eigenvalues of $RW$. It is shown that the density function $g(x)$ of $Q$ satisfies the integral equation $xg(x)=\int_0^x g(x-y)h(y)dy$ where $h(x)=\frac{1}{2}\sum \exp(-x/2\lambda_i)$. The details for a computational solution of this equation are described. The basic theorem on the asymptotic distribution of eigenvalues of Toeplitz forms [see U. Grenander and G. Szegö, *Toeplitz forms and their applications*, Univ. of Calif., Berkeley, 1958; MR **20** #1349] is used to obtain an approximate distribution when $R$ and $W$ are Toeplitz forms. Numerical examples comparing the exact distribution and the approximations proposed are given. *M. Rosenblatt* (Providence, R.I.)

1979:

**McGregor, J. R. An approximate test for serial correlation in polynomial regression.** Biometrika **47** (1960), 111–119.

The distribution of the von Neumann ratio for the residuals from a linear least squares regression varies with the regression vectors. Durbin and Watson [Biometrika **37** (1950), 409–428; **38** (1951), 159–178; MR **12**, 512; **13**, 144] found lower and upper bounds for the significance points. For polynomial regression, Hannan [Biometrika **44** (1957), 57–66; MR **19**, 333] showed that the true significance points agreed with the upper bounds to $O(n^{-2})$ where $n$ is the sample size. This paper derives an approximate distribution, in the general case, which gives significance points much nearer the upper than the lower bound. Hannan's result is verified. The method of steepest-descents is used, as in Daniels [Biometrika **43** (1956), 169–185; MR **18**, 79]. *G. S. Watson* (Toronto)

1980:

**Medhi, J. A note on the properties of a test procedure for discrimination between two types of spectra of a stationary process.** Skand. Aktuarietidskr. **1959**, 6–13.

## NUMERICAL METHODS

See also A1497, 1954, 2044, 2045, 2132.

1981:

**Михлин, С. Г. ★Вариационные методы в математической физике. [Mihlin, S. G. Variational methods in mathematical physics.]** Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1957. 476 pp. 16.75 rubles.

The book contains an introduction and eleven chapters. The introduction consists of a short essay on the development of variational methods in mathematical physics. The author gives a detailed account of the methods of Ritz and Bubnov-Galerkin. Chapter 1 is concerned with linear boundary problems as equations in $L_2$ space, with the necessary information about the latter. A knowledge on the part of the reader of the theory of abstract Hilbert spaces is not presupposed. Chapter 2 contains information about the various types of convergence in the mean and other auxiliary questions in the theory of functions. Chapter 3 deals with questions of solving an equation in the form $Au=f$, where $A$ is a formally self-adjoint positive-definite differential operator and $u$ and $f$ are the desired and given functions in $L_2$. This problem is made to depend (the energy method) upon the problem of finding the minimum of the corresponding quadratic functional

$$F(u) = (Au, u) - 2\,\mathrm{Re}\,(u, f).$$

An investigation is given of the Ritz method for constructing a minimizing sequence by means of a previously chosen system of coordinate functions. Other methods are also discussed for the construction of minimizing sequences, namely the method of Courant, the method of steepest descent, and the method of reduction to ordinary differential equations. Consideration is given to the special features that arise in the case of natural boundary conditions. The problems with non-homogeneous boundary conditions are also considered. The concepts of generalized derivative and function with finite energy are introduced. An investigation is given of the question of the existence of a solution of the variational problem without assuming the solvability of the original differential equation. In this connection the concept of generalized solution of the differential equation is introduced (the question is considered more fully in chapter 6).

In chapter 4 the author discusses the application of the energy method to various problems in mathematical physics. In particular, certain problems in the theory of elasticity are discussed, and also elliptic boundary problems with degeneration on parts of the boundary: The investigation reduces essentially to proof of the positive definiteness of the corresponding quadratic form and to a

more detailed study of the character of the convergence of the minimizing sequences. In chapter 5 the author, continuing the general arrangement of the two preceding chapters, obtains analogous results concerning the application of the energy method to eigenvalue problems. In all the cases considered he gives a proof of the discreteness of the spectrum. Convergence is proved for the Ritz method. The minimax principle of Courant is explained and some of its consequences are given.

The further development is based on the theory of operators in abstract Hilbert space (the spectral theory is not used). The necessary preliminary information is given in chapter 6. The Lebesgue integral and its fundamental properties are discussed. Hilbert space is treated axiomatically and the elements of its geometry are explained, together with the fundamental properties of linear operators and functionals. For a symmetric positive-definite operator $A$ the connection is explained between the solvability of the variational problem and the corresponding equation $Au=f$ in the case of the special extension, due to Friedrichs, of the operator $A$. The procedure for extension is described.

Chapter 7 contains an estimate of the error in the approximate solution obtained by the energy method. Approximation of this sort depends upon approximation from above and from below to the quantity $\inf F(u)$. In this connection various methods are considered for obtaining an approximation to $\inf F(u)$ from below: (1) The method of orthogonal projections (in particular the method of Castigliano in problems in the theory of elasticity); (2) the method of Trefftz for solution of the Dirichlet problem and its generalizations to other boundary problems; (3) Variants of the transformation of K. Friedrichs proposed by M. K. Slobodyanskiĭ. In the same chapter there is also a short consideration of questions of two-sided estimates of functionals of the form $(u, g)$, where $u$ is a solution of the equation $Au=f$ and $g$ is a given element, and of questions of two-sided estimation of eigenvalues and of the error arising from a mistake in the equation.

Chapter 8 contains numerical examples of calculations based on the method described in the earlier chapters. In most cases there is also an estimate of error. Indications are also given concerning the choice of coordinate functions for the construction of minimizing sequences. In chapter 9 there is a discussion of the Bubnov-Galerkin method. The method is described and there is also a simple proof of its convergence for integral equations and for one-dimensional differential boundary problems. The author explains the fundamental principles of the theory of completely continuous operators in Hilbert space and thereupon gives a proof for his general criterion for convergence of the Bubnov-Galerkin method. With the help of his criterion he investigates the convergence of the method for a series of elliptic boundary problems and eigenvalue problems. He touches upon the question of natural boundary conditions. He gives a numerical example. Chapter 10 gives a description and investigation of the convergence of the method of least squares, chiefly in the form proposed by the author. A general criterion of convergence for the method is given and applications are introduced to various boundary problems, among them certain problems in the theory of elasticity. The connection of this method with the energy method is noted. A numerical example is given. The chapter 11 "finite-difference method" does not claim to present a complete account of the matter. It only presents the solution of boundary problems by the method of nets. The straight-line method is considered in detail in its application to the harmonic and biharmonic problems for trapezoidal regions.

The book, which is based upon earlier work of the author, will be useful both for those who are interested in approximative calculation and also for mathematicians interested in the theory of direct methods.

*M. Š. Birman* (RŽMat 1958 #5813)

1982:

**Hunter, D. G. N. Note on a test for repeating cycles in a pseudo-random number generator.** Comput. J. **3** (1960/61), 9.

1983:

**Morduchow, Morris; Levin, Lionel. Comparison of the method of averages with the method of least squares: fitting a parabola.** J. Math. Phys. **38** (1959/60), 181–192.

The authors evaluate the ratio of the standard deviation, $\sigma_a$, of the residuals resulting from fitting a parabola to data at $n$ equally spaced points by averaging over subintervals $(n_1, n_2, n_3)$ to the s.d., $\sigma_s$, resulting from a least squares fit. For $(n/5, 3n/5, n/5)$, $n \neq 7$, $\sigma_a/\sigma_s < 6/5$ with a smaller asymptotic value valid for $n \geq 20$. Other interval choices are also discussed.

*M. E. Rose* (Livermore, Calif.)

1984:

**Kuntzmann, J. ★Méthodes numériques: Interpolation, dérivées.** Dunod, Paris, 1959. xvii+253 pp. 3.600 F.

Numerical methods for interpolation and differentiation are clearly presented with the aid of many diagrams, tables and illustrative examples. The author believes that this special work will help to fill the need for books in French on numerical methods.

Except for part of a fourteen page section on interpolation in the complex plane, the level of the book requires a knowledge of calculus (sometimes advanced and with determinants). The text is not intended as a specialized treatise. The type is exceptionally well set with a maximum of space and a minimum of print per page.

The chapter headings are: (1) Valeurs numériques de polynomes—différences; (2) Interpolation; (3) Compléments sur les tables; (4) Dérivées; (5) Arguments répétés: polynome défini par des valeurs des dérivées; (6) Fonctions d'une variable complexe: fonctions de plusieurs variables; (7) Théorie générale de l'interpolation par une famille linéaire; (8) Exemples simples d'interpolation par une famille non linéaire.

*E. Isaacson* (New York)

1985:

**Rivlin, T. J. A note on smooth interpolation.** SIAM Rev. **2** (1960), 27–30.

Let $P_m$ denote the set of all polynomials $P(x)$ of degree $m$, $m=n+k$, $k \geq 0$ which, interpolate to a given $f(x)$ at a given set of $n$ points: $P(x_i)=f(x_i)$ $(i=1, 2, \cdots, n)$. Set $s(P)=\int_{x_1}^{x_n}(1+P'(x)^2)^{1/2}dx$ and ask for $P^* \in P_m$ such that $s_m=s(P^*)=\min_{P \in P_m} s(P)$. The extremal $P^*$ exists and is unique [T. J. Rivlin, *Smooth interpolation*, SIAM Rev. 1

(1959), 60–63; MR **20** #6774]. Let $l$ designate the length of the polygonal line passing through the points $(x_i, f(x_i))$ $(i=1, 2, \cdots, n)$. The author now proves that $\lim_{m\to\infty} s_m = l$.

*P. J. Davis* (Washington, D.C.)

1986:

**Salzer, Herbert E. Hermite's general osculatory interpolation formula and a finite difference analogue.** J. Soc. Indust. Appl. Math. **8** (1960), 18–27.

Hermite's interpolation formula gives the unique polynomial of degree $\sum_{i=1}^{n} r_i - 1$ which, together with its first $r_i - 1$ derivatives at the distinct points $x_i$, $i=1, 2, \cdots, n$, assumes prescribed values. This paper gives a new derivation of the formula in the special case where the same number of derivatives is prescribed at each point $x_i$. This derivation leads to a form that is especially convenient for numerical computation. An expression is also given for the unique polynomial of degree $rn-1$ which agrees with a given function $f(x)$ and its first $r-1$ advancing differences at $n$ distinct points. Here the differences are for equally spaced arguments, though the $n$ points may be unequally spaced with reference to each other.

*T. N. E. Greville* (Kensington, Md.)

1987:

**Crossley, F. R. E.; Germen, Üstün. A method of numerical evaluation of a large determinant.** J. Appl. Mech. **27** (1960), 350–351.

This is the characteristic determinant of a particular tridiagonal matrix in which all row sums vanish except possibly the first and last. The algorithm is indeed neat, but it is not obviously superior to the usual recursion, especially if the matrix is symmetrizable, since then the terms of the recursion form a Sturm sequence.

*A. S. Householder* (Oak Ridge, Tenn.)

1988:

**Stiefel, E. Note on Jordan elimination, linear programming and Tchebycheff approximation.** Numer. Math. **2** (1960), 1–17.

The paper is concerned with the Tchebycheff "solution" of an overdetermined system of linear equations. It is shown that the algorithms suggested by Zuhovickiĭ [Dokl. Akad. Nauk SSSR **79** (1951), 561–564; MR **13**, 285] and the author [Numer. Math. **1** (1959), 1–28; MR **21** #6681] can be interpreted as applications of the simplex method of linear programming that correspond to dual programs. For a system of $n$ linear equations Zuhovickiĭ's method involves $2n$ linear inequality constraints, whereas the author's "exchange method" avoids this doubling effect. Moreover, there is no need of constructing a feasible solution to start the computations. To make the paper reasonably self-contained, the relations between Jordan elimination and the simplex method are discussed in detail before Tchebycheff's problem is attacked.

*W. Prager* (Providence, R.I.)

1989:

**Gastinel, Noël. Utilisation de matrices vérifiant une équation de degré 2 pour la transmutation de matrices.** C. R. Acad. Sci. Paris **250** (1960), 1960–1961.

A matrix whose minimal polynomial is quadratic is of the form $Z=aI+bK$, where $a$ and $b$ are scalars and $K$ is a matrix of rank 1. The author finds them useful in inversion and in normalization, as the reviewer had already shown [Householder, J. Soc. Indust. Appl. Math. **6** (1958), 189–195; MR **20** #2835; Householder and Bauer, Numer. Math. **1** (1959), 29–37; MR **20** #7387].

*A. S. Householder* (Oak Ridge, Tenn.)

1990:

**Boyd, K. T. Simultaneous equations and linear programming.** Comput. J. **3** (1960/61), 45–46, 50.

Author's summary: "A method is given by means of which a computer program for linear programming can be used to produce, in one run, the solution of a set of simultaneous linear equations and the inverse of the matrix of coefficients. The non-negativity rule usual in linear programming places no restriction on the problems that can be dealt with. The method can also be used for inversion of matrices not associated with simultaneous equations.

"Many programs have special features which will render parts of the procedure outlined superfluous, but these possibilities have been left out of account."

1991:

**Householder, Alston S. Generated error in rotational tridiagonalization.** J. Assoc. Comput. Mach. **5** (1958), 335–338.

Givens made an error analysis for his well-known tridiagonalization method for symmetric matrices in 1954. This analysis is quite laborious. The author of the present paper gives a considerably simpler method of analysis, which yields somewhat less precise error bounds.

*H. H. Goldstine* (Yorktown Heights, N.Y.)

1992:

**Householder, Alston S. Unitary triangularization of a nonsymmetric matrix.** J. Assoc. Comput. Mach. **5** (1958), 339–342.

The author analyzes a procedure for the inversion of non-symmetric matrices due to J. W. Givens and shows how the same result can be obtained with a substantial saving in the amount of work needed to be performed. In fact the number of square roots involved is reduced from $\sim n^2/2$ to $2n$, a saving of $\sim n^2/4$. The method of Givens is in actuality a way of triangularizing the matrix by plane rotations. It is thus very stable. The author's simplification is quite elegant and simple.

*H. H. Goldstine* (Yorktown Heights, N.Y.)

1993:

**Berman, Martin F. A method for transposing a matrix.** J. Assoc. Comput. Mach. **5** (1958), 383–384.

1994:

**Mysovskih, I. P. Estimate of the error of approximate methods of investigation of eigenvalues of a Hermite kernel.** Mat. Sb. (N.S.) **48 (90)** (1959), 137–148. (Russian)

The object is to give a priori bounds for the truncation error (i) when the kernel is replaced by a degenerate kernel, and (ii) when a quadrature formula is used. The estimates are made by means of a theorem of Weyl. A paper by Wielandt [Proc. Symposia Appl. Math., Vol. VI, pp. 261–282, Amer. Math. Soc., Providence, R.I., 1956; MR **19**, 179] to which reference is made, treats (ii) in much more detail, and (i) is trivial.

*A. S. Householder* (Oak Ridge, Tenn.)

1995:

**Wilkinson, J. H. Householder's method for the solution of the algebraic eigenproblem.** Comput. J. **3** (1960/61), 23–27.

This method [Householder and Bauer, Numer. Math. **1** (1959), 29–37; MR **20** #7387] uses a series of $n-2$ reflections to reduce a real symmetric matrix to tridiagonal form, where the method of Givens [Oak Ridge National Laboratory, Oak Ridge, Tenn., Rep. ORNL 1574 (1954); MR **16**, 177] uses $(n-1)(n-2)/2$ plane rotations. The author therefore finds the method more economical in arithmetic and programming, and somewhat more accurate. A computational procedure is described in detail, with a numerical example, an operational count is given, and an error analysis sketched. For nonsymmetric matrices the method leads to a Hessenberg form. For this the author prefers a somewhat simpler type of non-orthogonal transformation, but remarks that the greater numerical stability of the orthogonal transformations may be decisive for larger matrices.

*A. S. Householder* (Oak Ridge, Tenn.)

1996:

**Causey, Robert L. On some error bounds of Givens.** J. Assoc. Comput. Mach. **5** (1958), 127–131.

In describing his method for computing the characteristic roots of a real symmetric matrix [Oak Ridge National Laboratory, Oak Ridge, Tenn., Rep. ORNL 1574 (1954); MR **16**, 177], Givens gives a detailed error bound. The author finds, and corrects, a slight error in Givens's analysis, and finds that the final inequalities can even be sharpened slightly.

*A. S. Householder* (Oak Ridge, Tenn.)

1997:

**Šidlovskaya, N. A. Application of the method of differentiation with respect to a parameter to the solution of non-linear equations in Banach spaces.** Leningrad. Gos. Univ. Uč. Zap. Ser. Mat. Nauk **33** (1958), 3–17. (Russian)

Consider the equation $F(\lambda, x)=0$, where $\lambda$ is a real parameter, and $x$ and $F$ are points in a normed space. Suppose the solution $x=x(\lambda)$ is known to be $x_0$ for $\lambda=0$, and required at $\lambda=1$. With suitable assumptions, $F_x dx/d\lambda + F_\lambda = 0$, and if the linear operator $F_x$ has an inverse, the problem reduces to that of solving systems of linear differential equations with initial conditions $x(0)=x_0$. Davidenko [Dokl. Akad. Nauk SSSR **88** (1953), 601–602; Ukrain. Mat. Ž. **5** (1953), 196–206; MR **14**, 906; **15**, 164] has described this method for the solution of systems of ordinary equations, and the present author extends the method to more general spaces. This method and Newton's method may be usefully combined, since $x(0)$ is not required to be close to $x(1)$, whereas the computed $x(1)$ may require further improvement by Newton's method. A nonlinear boundary value problem is discussed by way of illustration.

*A. S. Householder* (Oak Ridge, Tenn.)

1998:

**Šisler, Miroslav. Die Konvergenz der Iterationsverfahren für die Lösung der Systeme nonlinearer Gleichungen.** Apl. Mat. **5** (1960), 141–150. (Czech. Russian and German summaries)

A system of equations $f(\mathbf{x})=0$ is to be solved by an iteration process $\mathbf{x}_{n+1}=\varphi(\mathbf{x}_n)$. A solution $\mathbf{x}=\mathbf{a}$ is a fixed point of the mapping $\varphi$ which is said to define an $N$th order iteration if all the partial derivatives up to the order $N-1$ of the components $\varphi_\mu$ are zero at $\mathbf{x}=\mathbf{a}$, but at least one of the order $N$ is different from zero. Sufficient conditions are established for the existence of a fixed point and the convergence of the iteration to a given order. There is a numerical example of two simultaneous cubic equations in two unknowns for which the generalized Newton method $\varphi(\mathbf{x})=\mathbf{x}-f(\mathbf{x})F^{-1}(\mathbf{x})$ yields an iteration of order 2; $F$ denotes the functional matrix of the first partial derivatives of the components of $f$.

*H. Schwerdtfeger* (Montreal)

1999:

**Abian, Smbat; Brown, Arthur B. On the solution of the equation $g(x)=0$.** Portugal. Math. **18** (1959), 101–106.

This note describes a kind of generalization of Newton's method for an equation $g(x)=0$ where $g(x)$ is continuous for $a-h\leq x\leq a+h$, but possibly non-differentiable. The corresponding iteration formula $x_n=\varphi(x_{n-1})$ such that $\xi=\lim x_n$ is a root of the given equation, is given by $\varphi(x)=x-mg(x)$ where $a$ is the initial value, $0<|g(a)|\leq Ch$, and $m$ a positive constant: $m(hD+|g(a)|)\leq 2h$; $C$ and $D$ are two positive constants, lower and upper bounds for the difference quotient of $g(x)$ in the interval mentioned above. An error estimate is provided by $|x_n-\xi|\leq hB^n$ where $B=1-mC$ or $=mD-1$. The reader who expects an application to a non-differentiable $g(x)$ will be disappointed. He gets only $g(x)=x^4-10$.

*H. Schwerdtfeger* (Montreal)

2000:

**Longman, I. M. A method for the numerical evaluation of finite integrals of oscillatory functions.** Math. Comput. **14** (1960), 53–59.

Let $S=v_0-v_1+v_2-\cdots+(-1)^n v_n$. The summation formula

$$S=(1/2)v_0-(1/4)\Delta v_0+(1/8)\Delta^2 v_0-\cdots$$
$$+(-1)^n[(1/2)v_n+(1/4)\Delta v_{n-1}+(1/8)\Delta^2 v_{n-2}+\cdots]$$

is developed and advocated for the rapid evaluation of sums where $v_i>0$ and increases slowly. Numerical examples are given for

$$\sum_{i=1}^{1000}(-1)^{i+1}i^{-1},\quad \sum_{i=1}^{9999}(-1)^{i+1}i^{1/2},\quad \int_0^{100\pi}(100^2\pi^2-x^2)^{1/2}\sin x\,dx.$$

*P. J. Davis* (Chevy Chase, Md.)

2001:

**Šaidaeva, T. A. Quadrature formulas with least estimate of the remainder for certain classes of functions.** Trudy Mat. Inst. Steklov. **53** (1959), 313–341. (Russian)

The author considers quadrature formulas of the form

$$\int_0^1 f(x)dx \simeq \sum_{k=0}^{n-1} p_k f(x_k).$$

Let $L$ denote a class of functions defined on the segment $[0, 1]$. A number which characterizes the precision of the formula for all functions in $L$ is

$$E_n(L; p_k, x_k) = \sup_f \left| \int_0^1 f(x)dx - \sum_{k=0}^{n-1} p_k f(x_k) \right| = \sup_f |R(f)|.$$

Denote $\min_{x_k, p_k} E(L; p_k, x_k)$ by $E(L)$. Quadrature formulas

for which the remainder $R(f)$ does not exceed $E(L)$ in magnitude are called formulas with least estimate of the remainder in the class $L$. Formulas with least estimate of the remainder are derived for the following classes of functions: (1) $|f''(x)| \leq k$ where no restrictions are placed on the nodes $x_k$; (2) $\{\int_0^1 |f^{(r)}(x)|^q dx\}^{1/q} \leq M_r$ $(r=1, 2; q>1)$; with no restrictions on the $x_k$; (3) $\mathrm{var}_{[0,1]} f^{(r-1)}(x) \leq M_{r-1}$ $(r=1, 2)$ with no restriction on the $x_k$, and for the case where the $x_k$ are equidistant.

*A. H. Stroud* (Madison, Wis.)

2002:

**Крылов, В. И. ★Приближенное вычисление интегралов. [Krylov, V. I. Approximate calculation of integrals.]** Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1959. 327 pp. 12.45 rubles.

Only simple integrals are considered, but one unusual feature of the book is a final "part" consisting of four chapters on indefinite integrals. The four chapters in the first part are entitled "Numbers and polynomials of Bernoulli", "Orthogonal polynomials", "Interpolation of functions", and "Linear, normed spaces. Linear operators". The remaining part of over 200 pages deals with definite integrals.

The topics dealt with in the introductory part are developed simply and easily, and only so far as they are required for the main purpose. The principal tool in the development of the quadrature formulas is the general remainder formula of much wider applicability that has been discussed in more general terms by Remež [Acad. Sci. RSS Ukraine. Rec. Trav. [Zbirnik Prace] Inst. Math. **1939**, no. 1, 21–62; **1940**, no. 4, 47–82; MR **2**, 195], by Sard [Duke Math. J. **15** (1948), 333–345; MR **10**, 197; and elsewhere], by Milne [J. Res. Nat. Bur. Standards **43** (1949), 501–511; MR **12**, 84], and others. Hitherto, this formula has not found its way to any extent into the textbooks [but see the reviewer's *Principles of numerical analysis*, McGraw-Hill, New York, 1953; MR **15**, 470] in spite of its power and basic simplicity. Here the author uses it skillfully to estimate error, and describes a technique for using it to improve the accuracy of a given formula.

The treatment is leisurely and elementary; tables of special coefficients are given. It is recommended highly.

*A. S. Householder* (Oak Ridge, Tenn.)

2003:

**Csoma, Zs. Angenäherte Quadratur im Falle ungleicher Teilintervalle.** Period. Polytech. Elec. Engrg. **4** (1960), 31–36.

Given the values $f_i = f(p_i)$ of the function $f(p)$ at the unequally spaced abscissas $p_i$ $(i=0, 1, \cdots, n=2k+1)$, the author derives the following generalized version of Simpson's formula:

$$\int_{p_0}^{p_n} f(p)dp = \tfrac{1}{3} \sum_{i=0}^{k} (g_{2i} + 4g_{2i+1} + g_{2i+2}) + H$$

$(g_i = \frac{1}{2} f_i(p_{i+1} - p_{i-1}))$. Two estimates for the error $H$ are presented.

*W. C. Rheinboldt* (Syracuse, N.Y.)

2004:

**Ihm, Peter. Numerical evaluation of certain multivariate normal integrals.** Sankhyā **21** (1959), 363–366.

It is shown that the probability integral of the multivariate normal distribution can be evaluated in terms of a simple integral of products of univariate normal integrals in the case where all the correlations are equal. The basic formula was previously obtained by M. R. Sampford as shown in the reviewer's paper [Proc. Cambridge Philos. Soc. **52** (1956), 230–233; MR **17**, 901]. It is suggested that the final integration be done numerically but it is not pointed out that such integration does in fact lead to exceptionally high accuracy if equidistant ordinates are used.

*P. A. P. Moran* (Oxford)

2005:

**Hsu, L. C. Concerning the numerical integration of periodic functions of several variables.** Acta Sci. Math. Szeged **20** (1959), 230–233.

A formula is obtained for the degree of approximation to the integral of a function $f(x_1, \cdots, x_m)$, of period $2\pi$ in each variable and with $p$ continuous partial derivatives $(p>3)$, by sums of the form $N^{-1} \sum_{k=1}^{N} f(2k\pi R^{-1}, \cdots, 2k\pi R^{-m})$, where $N = R^m$.

*P. Civin* (Gainesville, Fla.)

2006:

**Vilenkin, N. Ya. On approximate calculation of multiple integrals.** Vyčisl. Mat. **5** (1959), 58–71.

The author derives some simple results concerning approximate integration formulas for multiple integrals of the form

$$\int_D f(X)p(X)dX \simeq \sum_{s=1}^{m} A_s f(X_s),$$

where $X = (x_1, \cdots, x_n)$ is a point in $n$-dimensional Euclidean space and $D$ is a region in the space. He also derives certain formulas of low degree for cubes and spheres. The author seems unaware that most of his results are contained in papers by Tyler [Canad. J. Math. **5** (1953), 393–412; MR **15**, 67], Hammer and Wymore [Math. Tables Aids Comput. **11** (1957), 59–67; MR **19**, 323] and Hammer and Stroud [ibid. **12** (1958), 272–280; MR **21** #970].

*A. H. Stroud* (Madison, Wis.)

2007:

**Min, Szu-hoa. On the numerical integration of double and multiple integrals.** Sci. Record (N.S.) **3** (1959), 531–533.

The author states an error found for double quadrature formulas of the form

$$\int_0^1 \int_0^1 f(x, y)dxdy \doteq N^{-1} \sum_{\nu=0}^{N-1} f(x_\nu, y_\nu)$$

where the $x_\nu$, $y_\nu$ are determined by reversing the binary expansion of the integer $\nu$ and using the odd position bits for $x_\nu$, the even position bits for $y_\nu$.

The function $f$ is assumed to have period 1 in each $x$ and $y$ and to have continuous first order derivatives.

*P. C. Hammer* (Madison, Wis.)

2008:

**Pan, Cheng-tung. On the numerical integration of a kind of multiple integrals.** Sci. Record (N.S.) **3** (1959), 534–537.

The author states the following. Theorem: Let $f(x_1, \cdots, x_s)$ be a periodic function defined on the region $D$,

$$0 \leq x_1 \leq 1, \cdots, 0 \leq x_s \leq 1,$$

with period 1 with respect to each of the variables. If $f(x_1, \cdots, x_s) \in C^m(D)$ $(m \geqq 0)$, i.e., if $f(x_1, \cdots, x_s)$ has continuous partial derivatives of order $m$ on $D$, and if $\xi_\nu(k) = \{k\alpha^{\nu-1}/N\}$, $(\nu = 1, \cdots, s)$, where $N$ is a positive integer and $\alpha$ is any integer lying between $\frac{1}{2}N^{1/s}$ and $N^{1/s}$, then

$$\int_0^1 \cdots \int_0^1 f(x_1, \cdots, x_s)dx_1 \cdots dx_s = \frac{1}{N}\sum_{k=1}^{N} f(\xi_1(k), \cdots, \xi_s(k)) + O\left(\frac{1}{N^{m/s}}\,\omega_m\left(\frac{3}{N^{1/s}}\right)\right),$$

where $\omega_m(\rho) = \max_{\alpha_1+\cdots+\alpha_s=m;\alpha_i\geq 0} \omega_{\alpha_1,\cdots,\alpha_s}(\rho)$, and $\omega_{\alpha_1,\cdots,\alpha_s}(\rho)$ denotes the modulus of continuity of the function

$$\frac{\partial^m f(x_1 \cdots x_s)}{\partial x_1^{\alpha_1} \cdots \partial x_s^{\alpha_s}} \quad (\alpha_i \geqq 0, \alpha_1 + \cdots + \alpha_s = m).$$

*P. C. Hammer* (Madison, Wis.)

2009:

**Ermakov, S. M. On a method of construction of cubature formulas.** Trudy Mat. Inst. Steklov. **53** (1959), 37–41. (Russian)

The author considers cubature formulas of the form

$$\iint_D w(x, y)f(x, y)dxdy \simeq \sum_{i=1}^{N} A_i f(x_i, y_i),$$

where $D$ is a domain in the Euclidean plane and where the formula is supposed to be exact for all polynomials of less than or equal to a certain fixed degree. He shows (1) for an arbitrary $D$ how a formula of degree $k+2$ may be obtained, with $w(x, y) \equiv 1$, providing a formula of degree $k$ is known with $w(x, y) = xy$, and (2) if $D$ is centrally symmetric how a formula of degree $2k+5$ may be obtained, with $w(x, y) \equiv 1$, providing a formula of degree $k$ is known with $w(x, y) = x^2y^2$. An example is given of a formula of degree 9 for the square with vertices $(\pm 1, \pm 1)$ which was constructed from a formula of degree 3. *A. H. Stroud* (Madison, Wis.)

2010:

**Solodov, V. M. Computation of iterated integrals.** Dokl. Akad. Nauk SSSR **127** (1959), 753–756. (Russian)

The author proves a number of results approximating to multiple integrals by finite sums, of which the following is the simplest. If the function $f(x_1, x_2, \cdots, x_s)$ satisfies the Lipschitz condition

$$|f(x_1, \cdots, x_s) - f(x_1', \cdots, x_s')| \leqq C\rho^\alpha$$

for $\alpha \leqq 1$ and $(x_1 - x_1')^2 + \cdots + (x_s - x_s')^2 \leqq \rho^2$, then

$$\left| \int_0^1 \cdots \int_0^1 f(x_1, \cdots, x_s)dx_1 \cdots dx_s - N^{-1} \sum_{n=1}^{N} f(M_n) \right| \leqq (s-1)\sigma/\sqrt{N} + CA/N^\alpha,$$

where $A$ depends only on $s$, $N > s$, and

$$M_n = \left(\frac{n}{N}, \frac{n^2}{N}, \cdots, \frac{n^s}{N}\right).$$

The quantity $\sigma$ is an upper bound for the sum of the absolute values of the Fourier coefficients of $f$. The author compares his results with those of M. Cheng and Y. Chen [Acta Sci. Nat. Univ. Pekinensis (4) **1956**, 411] and L. C. Hsu and L. W. Lin [Sci. Record **2** (1958), 215–219].

*R. A. Rankin* (Glasgow)

2011:

**Conte, S. D.; Miles, J. W. On the numerical integration of the Orr-Sommerfeld equation.** J. Soc. Indust. Appl. Math. **7** (1959), 361–366.

The problem considered here arose in connection with the second author's previous work on the generation of surface waves [J. Fluid Mech. **3** (1957), 185–204; MR **19**, 1004]. In that work it was shown that the energy-transfer coefficient which describes the transfer of energy from the shear flow to the surface waves could be determined from the solution of the inviscid Orr-Sommerfeld equation with a logarithmic mean velocity distribution. In treating this problem numerically, some care is required because of the presence of a logarithmic branch point in one of the solutions. A method for dealing with this situation is described that is suitable for high-speed, numerical computation and some numerical results are given. *W. H. Reid* (Providence, R.I.)

2012:

**Il'in, V. P. Estimate of error in the Ritz method for ordinary differential equations.** Trudy Mat. Inst. Steklov. **53** (1959), 43–63. (Russian)

The author develops by elementary methods a number of a posteriori error estimates in the approximate solution of boundary value problems for linear self-adjoint differential equations. The following is a typical result. Consider

$$L(y) \equiv -(d/dx)(pdy/dx) + qy = f, \quad y(0) = y(1) = 0,$$

where $q, f \in C$, $p \in C^1$, $p(x) \geqq \tilde{p} > 0$, $q(x) \geqq 0$ in $[0, 1]$. Let $y_n \in C^2$ be an approximation (furnished, e.g., by the Ritz method) to the exact solution $y$ satisfying $y_n(0) = y_n(1) = 0$. Set $f_n = L(y_n) - f$. Then

$$\max_{x\in[0,1]} |y_n(x) - y(x)| \leqq \frac{1}{2}\left[\int_0^1 \frac{dx}{p(x)}\right]^{1/2}\left[\int_0^1 \frac{dx}{p(x)}\left(\int_0^1 f_n(t)K(t, x)dt\right)^2\right]^{1/2}$$

where $K(t, x) = -P(t)/P(1)$ if $0 \leqq t < x$, $K(t, x) = 1 - [P(t)/P(1)]$ if $x < t \leqq 1$, $P(t) = \int_0^t ds/p(s)$. From this result various simpler but weaker estimates are derived including one by Tatarkiewicz [Ann. Polon. Math. **1** (1955), 346–359; MR **17**, 539]. Further variants of these estimates are obtained under different assumptions on $p$ and $q$, and also for other boundary conditions, such as $y'(0) = y'(1) = 0$, and $y'(0) - h_1y(0) = y'(1) + h_2y(1) = 0$, $h_1 > 0$, $h_2 > 0$. There is also a remark on error estimates for the first derivative. Finally, a special boundary value problem for self-adjoint differential equations of higher order is considered. A few numerical examples illustrate the quality of the error bounds obtained. *Walter Gautschi* (Oak Ridge, Tenn.)

2013:

**Krogdahl, Wasley S. Numerical solutions of the Van der Pol equation.** Z. Angew. Math. Phys. **11** (1960), 59–63. (German summary)

The van der Pol equation

$$\ddot{x} + \lambda(x^2 - 1)\dot{x} + x = 0$$

is well known to have a (unique) limit-cycle $\gamma_\lambda$ for every value of $\lambda \geqq 0$. The period, amplitude and certain other parameters of $\gamma_\lambda$ are known asymptotically for both very

large and very small values of $\lambda$. This describes a computation (on IBM 650, Northwestern University) of the properties of $\gamma_\lambda$ for integral values of $\lambda$ from 1 to 10 (inclusive). Tables are given of the values obtained and comparison made with known asymptotic values.

References: Gill, Proc. Cambridge Philos. Soc. **47** (1951), 96–108 [MR **12**, 538]; Lefschetz, Nat. Bur. Standards Appl. Math. Ser. No. 15 (1951); Minorsky, *Introduction to non-linear mechanics*, Edwards, Ann Arbor, Mich., 1947 [MR **8**, 583], pp. 113–115; Andronov and Haĭkin, *Theory of oscillations*, Princeton Univ. Press, 1949 [MR **10**, 535], p. 143; Stoker, *Nonlinear vibrations*, Interscience, 1950 [MR **11**, 666], p. 134; Dorodnicyn, Akad. Nauk SSSR. Prikl. Mat. Meh. **11** (1947), 313–328 [MR **9**, 144]. *S. Lefschetz* (Mexico City)

2014:

**Cole, R. W. A note on numerical solution of certain linear boundary value problems.** J. Assoc. Comput. Mach. **5** (1958), 258–260.

The author describes a method—equivalent to Euler's "Taylor-series method"—for the approximate solution of a system of linear ordinary first order differential equations. No reference to the literature is made.

*W. C. Rheinboldt* (Syracuse, N.Y.)

2015:

**Položiĭ, G. N. A numerical method of solving boundary value problems for partial differential equations.** Dokl. Akad. Nauk SSSR **134** (1960), 39–41 (Russian); translated as Soviet Math. Dokl. **1** (1961), 1016–1019.

Dirichlet's problem for the usual finite difference Laplace operator can be explicitly solved for a rectangle whose sides lie on grid lines of the finite difference mesh. Dirichlet's problem for a region $G$ which is the union of such rectangles with disjoint interiors is considered. The explicit solution for each rectangle leads to a system of linear equations for the values of the solution at the mesh points of the common boundaries of rectangles. The number of such points may be significantly smaller than the total number of interior points of $G$. (The method is closely related to block relaxation [R. V. Southwell, *Relaxation methods in theoretical physics*, Clarendon, Oxford, 1946; MR 8, 355].)

The same method also applies to biharmonic and other finite difference problems.

*H. F. Weinberger* (Minneapolis, Minn.)

2016:

**Dames, Ralph T. Stability and convergence for a numerical solution of the Goursat problem.** J. Math. Phys. **38** (1959/60), 42–67.

The Goursat problem for the system of hyperbolic equations

$$u_{xy} + Au_x + Bu_y + Cu = s$$

is treated numerically, with the boundary conditions:

$$u(x, +0) = f(x) \quad \text{for } 0 \leq x \leq X,$$
$$u(+0, y) = g(y) \quad \text{for } 0 \leq y \leq Y,$$

with $f(0) = g(0)$. The vectors $u, f, g, s$ have $p$ components and the matrices $A, B, C$ are $p$ by $p$. The linear case in which $A, B, C$ and $s$ are functions of $(x, y)$ is dealt with first and afterwards the quasi-linear case in which their arguments are $(x, y, u)$ is handled.

The stability and convergence of a simple, explicit finite difference scheme is first established for the linear case and then the quasilinear case. In the linear case this analysis is based on the corresponding numerical treatment of the equivalent first order system. The author also treats the pure initial value problem and the mixed initial boundary value problem. *E. Isaacson* (New York)

2017:

**Conway, H. D. The approximate analysis of certain boundary-value problems.** J. Appl. Mech. **27** (1960), 275–277.

2018:

**Makino, Ray. Numerical integration of the equations of unsteady plane, cylindrical and spherical flows.** Ordnance Comput. Res. Rep. **5** (1958), 1–9.

The paper gives a practical discussion of how to set up finite difference schemes for the characteristic form of the equations. Both the Lagrange and Eulerian form are treated. *E. Isaacson* (New York)

2019:

**Douglas, Jim, Jr. On the numerical solution of a nonlinear, first-order differential equation.** Symposium on the numerical treatment of partial differential equations with real characteristics: Proceedings of the Rome Symposium (28–29–30 January 1959) organized by the Provisional International Computation Centre, pp. 12–16. Libreria Eredi Virgilio Veschi, Rome, 1959. xii + 158 pp.

The initial value problem for the equation $u_t = g(u)u_x$ for $-\infty < x < \infty$, $t > 0$, where $u(x, 0) = f(x)$, $-\infty < x < \infty$, is treated numerically. Under suitable assumptions on $f$ and $g$ the author proves the convergence of a simple explicit finite difference scheme for the case in which the solution is discontinuous. *E. Isaacson* (New York)

2020:

**Keller, Herbert B. Approximate solutions of transport problems. II. Convergence and applications of the discrete ordinate method.** J. Soc. Indust. Appl. Math. **8** (1960), 43–73.

The author continues his previous research [same J. **6** (1958), 452–465; MR **21** #5278] on approximating solutions of the transport equation. The monoenergetic neutron transport equation in plane geometry for an isotropically scattering homogeneous medium can be written as

$$(*) \quad \mu \frac{\partial \Phi(x, \mu)}{\partial x} + \sigma \Phi(x, \mu) = \sigma \frac{c}{2} \int_{-1}^{+1} \Phi(x, \mu) d\mu + S(x, \mu),$$

where $\Phi(x, \mu)$ is the neutron flux at position $x$ whose velocity vector makes an angle $\cos^{-1} \mu$ with the positive $x$ axis, $\sigma$ is the total macroscopic cross section, $c$ is the average number of secondary neutrons produced per collision, and $S(x, \mu)$ is an inhomogeneous source of neutrons. Basically, there are two well-known procedures for approximating the solution of this transport equation, one based on expansions in orthogonal functions of $\mu$

such as Yvon's method, and the other based on discrete ordinates in $\mu$, and there is a certain equivalence between these methods. The author then investigates the discrete ordinate method, and he does this in a sufficiently general way so that all reported variations of the discrete ordinate method which are in use are analysed. His basic result is a mean-square convergence theorem of the discrete ordinate methods to the solution of (*). The proof is valid for arbitrary nonhomogeneous media.

Concentrating on the case of a homogeneous slab, the general solution of the discrete ordinate approximation is obtained by means of matrix calculus. For uniform sources, this general solution simplifies, and gives rise to a practical method of computation.

Finally, for homogeneous gray slabs, an approximation for the neutron capture fraction is given, and asymptotic representations are obtained for very thick or very thin slabs. Also, the critical equation is derived and represented in a form which allows a simple numerical determination of the critical half-thickness of the slab.

*R. S. Varga* (Cleveland, Ohio)

2021:

**Borisov, S. N. On nomograms with tangent contact for certain empirical dependences.** Vyčisl. Mat. **5** (1959), 79–82.

The author treats the case, occurring often in practice, when the empirical relationship in the first approximation is $z=Ax^{\alpha}y^{\beta}$, $A, \alpha, \beta$ constants. He shows then a nomogram with tangent contacts for the formula for pressure loss per current meter during the flow of a liquid or a fluid in pipes. *D. Mazkewitsch* (Cincinnati, Ohio)

2022:

**Lapteva, D. G. On projective transformation of alignment nomograms with rectilinear answer scale.** Vyčisl. Mat. **4** (1959), 150–158. (Russian)

Usually homology is applied for a projective transformation, which may be written

$$(*) \qquad x_1 = bx/(y+b), \quad y_1 = (L+b)y/(y+b),$$

where $L$ is a constant and $b$ the transformation parameter. The greatest difficulty in a transformation is the selection of $b$. Practically it is important to know those values of $b$ for which the scale approximates the requested type. From (*) one gets

$$(L-y_1)/y_1 = (L-y)/y \cdot b/(L+b),$$

$$\log(L-y_1)/y_1 = \log(L-y)/y + \log b/(L+b).$$

For this equation one may construct an addition nomogram with a uniform answer scale for $y_1$ and scales for $y$ and $b$ situated on an ellipse. The nomogram gives then a clear picture of the mutual relation between $y$, $y_1$ and $b$, and permits an easy estimate of the influence of the parameter $b$ on the character of the answer scale.

*D. Mazkewitsch* (Cincinnati, Ohio)

2023:

**Denisyuk, I. N. On effective formulas for projective transformation and their application to construction of empirical equations.** Vyčisl. Mat. **4** (1959), 162–166. (Russian)

In section 1 the author gives an analytical expression for the parameter $m$ which characterizes the "spectrum" of the projectively transformed scale. The parameter enters in formulas given in his article in Nomograf. Sb. **1935**, 201–217. In section 2, for the case that the curve is rectifiable on logarithmic or semi-logarithmic paper, formulas are given enabling one to write down the relationship in the form $y=a+b/(x+c)$.

*D. Mazkewitsch* (Cincinnati, Ohio)

2024:

**Smorkačev, E. T. The plotting of local nomographs without recalibration.** Dokl. Akad. Nauk SSSR **129** (1959), 515–518. (Russian)

The author gives a practical method for plotting the local nomographs without recalibration with accuracy to the infinitesimal of the sixth order. It is based on the theory developed by Kreĭnes and Aĭzenštat [Mat. Sb. (N.S.) **37 (79)** (1955), 337–352; MR **17**, 1010]. Explicit formulas for calculating the first few coefficients required for plotting the nomographs are included.

*S. Kulik* (Long Beach, Calif.)

2025:

**Smorkačev, E. T. The plotting of local nomographs without permissible transformations.** Dokl. Akad. Nauk SSSR **129** (1959), 1242–1245. (Russian)

This note is a continuation of the previous paper by the author [see preceding review]. It presents the explicit formulas for the last group of the coefficients required for plotting nomographs without recalibration.

*S. Kulik* (Long Beach, Calif.)

2026:

**Hovanskiĭ, G. S. Some questions in practical nomography.** Vyčisl. Mat. **4** (1959), 3–103. (Russian)

From author's summary: "The paper presents new methods of constructing nomograms satisfying practical demands; it is divided into six chapters. The first three chapters represent further development of the author's previous papers. The fourth chapter considers from a general point of view various methods of approximate representation of functions of one variable, based on use of nomograms with three degrees of freedom of motion of the movable plane, of nomograms with oriented plane, and of alignment nomograms. The fifth chapter gives a method of construction of approximate alignment nomograms with coinciding parallel scales for two equations of Cauchy type. The sixth chapter presents ways of application of nomograms for investigation of functional relations." *D. Mazkewitsch* (Cincinnati, Ohio)

2027:

**Lapteva, D. G. On construction of a nomogram for a system of equations $f_3(v)=f_1(u)+f_2(w)$; $f_2(v)=\varphi_1(u)+f_4(t)$ with three point contacts and one tangent contact.** Vyčisl. Mat. **5** (1959), 133–140. (Russian)

In the above system of equations $w$ and $t$ are assumed given, $u$ and $v$ are requested. The above system may always be represented by a nomogram with three point contacts and one tangent contact. The author gives a method for constructing nomograms of this type. As an example is given the construction of the nomogram for

$$Q = (bh+mh^2)v, \quad v = C\sqrt{(Ri)},$$

which are equations used in hydraulic calculations of trapezoidal channels. *D. Mazkewitsch* (Cincinnati, Ohio)

2028:

**Gloden, A. Table de factorisation des nombres $N^4+1$ pour l'intervalle 6001–7000. Chiffres 2 (1959), 209–218.** (English, German and Russian summaries)

The table is as described in the title. Incompletely factored entries have missing prime factors $>10^6$. This is an extension of previous tables of Cunningham [*Binomial factorisations*, Vol. I, Hodgson, London, 1923; pp. 113–119] and the author. *D. H. Lehmer* (Berkeley, Calif.)

2029:

**Казинский, В. А. ★Математические таблицы для аппроксимации геофизических аномалий и редукций интерполяционными многочленами. [Kazinskiĭ, V. A. Mathematical tables for approximation of geophysical anomalies and reductions by interpolatory polynomials.]** Akademiya Nauk SSSR, Institut Fiziki Zemli. Izdat. Akad. Nauk SSSR, Moscow, 1959. 90 pp. 5.75 rubles.

The nine tables and one nomogram are designed to help the geophysicist compute the effect on the Newtonian gravity field of a large mass anomaly of irregular shape located in the earth's interior. Let $\rho=(x^2+y^2+z^2)^{-1/2}$. The largest table gives $10^{-3}/\rho$ to the nearest unit (about 3 significant decimals) for $x=0(.25)\ 3(.5)\ 6,\ 10,\ 15,\ 20,\ 25,\ 50,\ 100,\ 200,\ 500$, for approximately 20 values (depending on $x$) of each of $y$ and $z$. Other tables give $x/\rho^3$, $z/\rho^3$, $y/\rho^3$, and various functions like $xy(x^2+z^2)^{-2}(y/\rho-y^3/3\rho^3)$. There are also short tables relating the geoid to the spheroid.

There is an 18-page explanation of the use of the tables in geophysical calculations, but no indication of the accuracy or method of construction of the tables. The typography is not very good.

*G. E. Forsythe* (Stanford, Calif.)

2030:

**★Бурунова, Н. М. Справочник по математическим таблицам. Дополнение №. 1. [Burunova, N. M. Reference book on mathematical tables. Supplement No. 1.]** Akademiya Nauk SSSR. Vyčislitel'nyĭ Centr. Izdat. Akad. Nauk SSSR, Moscow, 1959. xxxix+183 pp. 9 rubles.

This is a welcome supplement to Lebedev and Fedorova's bibliography of mathematical tables [*Spravočnik po matematičeskim tablicam*, Izdat. Akad. Nauk SSSR, Moscow, 1956; MR 18, 828—see the review for background]. Whereas LF [Lebedev and Fedorova] covers tables available up to about 1954, B [Burunova] claims to extend the coverage to material contained in certain Soviet libraries or reviewed in the world's principal abstracting journals before approximately the middle of 1958.

The publication dates for tables cited in B actually range from 1942 to 1959, but most are from 1951 to 1956.

The format of Burunova follows LF exactly. B's outline of functions is less complete than LF's in that B includes only titles not in LF. However, B includes a table of contents to the outline of functions in both LF and B, with page numbers for LF and B in parallel columns. The functions listed in B but not in LF are: logarithms to base 2, confluent hypergeometric functions, the Riemann $\zeta$-function, biharmonic polynomials, Weber functions, and functions related to light-scattering.

On page 180 of the index, a block of author names is out of order. *G. E. Forsythe* (Stanford, Calif.)

## COMPUTING MACHINES

See also 2132, 2479, 2505.

2031:

**Young, Frederick H. Analysis of shift register counters.** J. Assoc. Comput. Mach. 5 (1958), 385–388.

This brief paper deals with some theorems concerning cycle lengths associated with a given number of flip-flops in a shift register counter. *A. A. Mullin* (Urbana, Ill.)

2032:

**Zuse, Konrad. Einige Gesichtspunkte der Entwicklung programmgesteuerter Rechenanlagen in den letzten 20 Jahren.** Allg. Statist. Arch. 43 (1959), 334–340.

This article, an expository one aimed at statisticians, is mainly of interest because of its author, who carried on independent digital computer design and construction during the last war in Germany. He reports briefly on computer history there and in the U.S.A., and then gives a description of present-day digital computers and their usage. This article represents European views on computers that might be of interest to a U.S. reader who has heard only one version.

*J. E. Carr, III* (Chapel Hill, N.C.)

2033:

**Blumenthal, Sherman. A dual master file system for a tape processing computer.** J. Assoc. Comput. Mach. 5 (1958), 319–327.

In standard file processing master-file records (usually on magnetic tape) are sequenced according to some key, and during the processing these records are compared with those of an activity file sequenced in the same way. This process can be wasteful of computer time when the number of records in the activity file is a small fraction of those in the master file.

In the system proposed here the master file consists of two files in one-to-one correspondence. The first is a smaller file containing only an abstract of the records in the larger file, and it is frequently run. The larger file is run at less frequent intervals. Savings result because of the shorter processing time for the smaller file. The small file need not even be complete. Records which are used only occasionally can be culled and transaction items corresponding to such records can be processed as if they were errors. A criterion for culling based on the history of activity and the cost of handling errors is suggested.

*C. C. Gotlieb* (Toronto)

2034:

**Cleave, J. P. Algorithms for formula translation.** Comput. J. 2 (1959), 53–54.

The author suggests the extension of formula translation in automatic programming to include quantities defined implicitly by formulae. From his examples one presumes the following conditions: (1) The variable for the implicitly defined quantity appears only once; (2) in the solution process each component function or operator has a unique inverse.

It appears to the reviewer that the applicability of such a plan is so restricted as to make it extremely easy for the programmer to express the quantity explicitly in

less time than the automatic translator would require; especially, if that translator is to detect automatically errors due to the fact that the explicit solution is either non-existent or non-unique. *S. Gorn* (Philadelphia, Pa.)

2035:

**Cockayne, A. H.; Hyde, E. Prime number coding for information retrieval.** Comput. J. **3** (1960/61), 21–22.

A direct way to examine whether organic chemical compounds have specified structural constituents is to assign each compound a number in which each bit position corresponds to one of the constituents. Constituents are then recognized by the presence of ones in the appropriate positions. In the alternative method, described here, each distinct structural constituent of the chemical compound is identified by a prime number, and the compound is given the identification number formed from the product of its constituent primes. Any compound can then be examined for the presence of a set of constituents by examining whether the corresponding product of primes divides the identification number.

In the application given there are 208 structural constituents, fewer than 10 of these being present in most compounds. The compound identification numbers could be stored in a double-length computer word of 76 bits resulting in an appreciable saving in storage over the first method which would require 208 bits for each compound. The time for determining whether a given product of primes divides the identification number is comparable to the time that would be required to preform an extraction by the first method. *C. C. Gotlieb* (Toronto)

2036:

**★Жоголев, Е. А.; Росляков, Г. С.; Трифонов, Н. П.; Шура-Бура, М. Р. Система стандартных подпрограмм. [Žogolev, E. A.; Roslyakov, G. S.; Trifonov, N. P.; Šura-Bura, M. R. A system of standard subprograms.]** Edited by M. R. Šura-Bura. Biblioteka Prikladnogo Analiza i Vyčislitel'noĭ Matematiki. Gosudarstv. Izdat. Fiz-Mat. Lit., Moscow, 1958. 231 pp. 6.10 r.

A system of standard subroutines for the Soviet computer M-2 (Strela?) at the Moscow State University Computation Center is described in very thorough detail, in fact the most thorough of any book on computers since the earlier works by Wilkes, Wheeler, and Gill on the EDSAC. There is a very detailed description of the machine M-2, which has a cathode-ray-tube storage, and of basic programming for that machine. There is a description of the method of logical program schemes (apparently first due to Lyapunov) and of how standard subprograms may be used with it. The remainder of the book is a detailed description of the subroutines for floating point operation and separately for fixed point operation, including the standard functions, input-output, and number conversion in both cases. Finally subroutines for Runge-Kutta numerical integration of ordinary differential equations, Simpson's rule, and solution of systems of linear equations are included.

The work described was apparently done in 1955–1956. There appears to be no major contribution concerning methods of approximations, etc., over what has been done in computation centers in the United Kingdom and U.S.A. up to that time. Nevertheless, the book deserves recognition as a very thorough documentation of the work of one computation center using a specific machine. *J. W. Carr, III* (Chapel Hill, N.C.)

2037:

**Samuel, A. L. Some studies in machine learning using the game of checkers.** IBM J. Res. Develop. **3** (1959), 211–229. (2 inserts)

The author describes his checker-playing program for the IBM-704 and its adaptation into a learning routine. The main model for the game is essentially that of Turing's modification of Shannon's model for chess, as described by Newell, Shaw, and Simon [same J. **2** (1958), 320–335; MR **21** #446], namely the search through a 'tree' of valid positions, extended in each branch beyond a depth of three to an essentially 'static' position. This last has a graded definition up to a depth of twenty. The selection method, as in the chess model, is the evaluation from the branch ends back to the root (the present position) of a function at each node by successive minimaxing over the branches emanating from that node. The evaluation function is considerably more complicated than Turing's 'material value'. It is a polynomial in a selection from some thirty-eight quantities measuring such things as 'advancement', 'denial of occupancy', 'total mobility', etc. Some of these quantities are attached to Boolean combinations of such measures as 'undenied mobility', and 'center control', so that the resulting evaluation polynomial cannot be considered linear.

The author applies two methods to make his program self-modifying in such a way as to simulate 'learning'. They correspond roughly to the reviewer's classification of learning models into the 'explicit' and 'implicit' types [Information and Control **2** (1959), 226–259; MR **21** #7608]. The explicit type, called 'rote learning', stores previously examined board positions and their values; whatever is saved by use of 'look-up time' can be used to evaluate other possibilities or to add depth to the values of those examined. The implicit type, called 'generalization' by the author, reinforces, not by modifying probabilities, as is usual among psychologists, but by modifying the coefficients of the evaluation polynomial.

Attached to these models are subroutines to permit games with people at the console, comparison with and learning from book games, games between sections of the machine, and general statistics gathering.

From the author's detailed description of the various instabilities, and insensitivities of the program, the reviewer infers that the model has a basic inflexibility in its use of the evaluating function. The coefficients are being 'learned' as though they should be constants throughout any one game. As a matter of fact ordinary 'center control' is much more important at the beginning of the game than toward the end, where the reverse is true for 'king's center control'. The evaluation criterion should be essentially a function of time, or at least 'phase' of the game. The learning procedure described, by not making such a distinction, effectively cancels out in one part of a game what it learns for another. It is for reasons such as this that the model in the cited reference by Newell, Shaw, and Simon was designed without a unique evaluation function.

It is a tribute to the author's careful marshalling of facts and statistics about the behavior of his program, in

the true scientific tradition, that points of criticism in such a complex program can even become apparent.

*S. Gorn* (Philadelphia, Pa.)

2038:

**Oettinger, Anthony G. Programming a digital computer to learn.** Methodos **11** (1959), 55–79.

This article is a reprint of Philos. Mag. (7) **43** (1952), 1243–1263 [MR **14**, 587].

2039:

**Papworth, D. G. Computers and change-ringing.** Comput. J. **3** (1960/61), 47–50.

Author's summary: "This paper describes some problems of change-ringing on church bells, and gives details of their solution on a Pegasus."

2040:

**Tillitt, Harley. Computer programming for young students.** J. Assoc. Comput. Mach. **5** (1958), 309–318.

2041:

**Leah, F. Methods of simulating a differential analyzer on a digital computer.** J. Assoc. Comput. Mach. **5** (1958), 281–288.

Logical functions of an analogue computer are simulated on a digital computer through an interpretative program using a Runge-Kutta fourth order integration scheme.

*M. E. Rose* (Livermore, Calif.)

2042:

**De Backer, W. Error analysis of a D-C integrator.** Ann. Assoc. Internat. Calcul. Anal. **2** (1960), 13–23.

The errors in the standard type of D.C. integrator, which uses a vacuum tube and feed back condenser, are shown to fall into the following four classes: (i) finite gain; (ii) drift; (iii) noise; (iv) grid-current. By analysing the effect of these, some suggestions are made regarding the best way to 'programme' an analogue computer for maximum accuracy. Numerical examples are given which show that accuracies of $\frac{1}{10}\%$ can be obtained using computing elements of ordinary characteristics so long as care is taken to work out the optimum set-up.

*A. D. Booth* (London)

2043:

**Harbert, F. C. The generation of Fourier transforms and coefficients on an analogue computer.** Electronic Engrg. **32** (1960), 496–498.

2044:

**Su, Hsuan-Loh. Electric analog for theory of adjustment and regression.** J. Math. and Phys. **38** (1959/60), 312–326.

The author discusses again the known fact that there exists an electric network analog to any physical or mathematical system which is describable by a set of linear equations, which are derivable by the minimization of a quadratic form, provided negative circuit elements are available. By way of illustration, the author shows how a least squares fit is made to a set of ten surveying equations. It is pointed out that in addition to the speed and efficiency with which a good approximation can be obtained by analog methods, the network approach to the problem is suggestive of iterative numerical procedures which are capable of achieving any desired accuracy of solution. A current distribution method is worked out in detail.

*H. M. Trent* (Washington, D.C.)

2045:

**Boscher, Jean. Exploitation des effets élémentaires dans l'analogie des réseaux résistifs.** C. R. Acad. Sci. Paris **250** (1960), 448–450.

The paper is concerned with exploiting the techniques of influence coefficients in solving Laplace type partial differential equations on a resistance network analyser. It is applicable to problems in which values and/or derivatives of the solution are required on the boundary only.

An equation

$$\frac{\partial}{\partial x}\left(m\frac{\partial W}{\partial x}\right)+\frac{\partial}{\partial y}\left(n\frac{\partial W}{\partial y}\right)=0$$

($m$, $n$ functions of $x, y$) holding over a finite domain $D$, bounded by a contour $C$, is simulated on a network. The technique is concerned solely with measurements at nodes of the network lying on $C$. In turn at each such node, $p_i$ ($i=1, \cdots, n$) say, a unit potential is applied with all other nodes held at zero potential; the values of the partial derivatives $(\partial W/\partial x)_{ij}$, $(\partial W/\partial y)_{ij}$ ($j=1, \cdots, n$) are then measured. Linear superposition then enables $(\partial W/\partial n)$ to be obtained for any given values, $W_i$, of $W$ on the boundary $C$ (the Dirichlet problem). Problems involving values of derivatives of $W$ in the boundary conditions can be dealt with by solving a set of linear equations. The author suggests the technique may be used where the differential equation is solved by relaxation methods.

The reviewer is sceptical of the general value of this process; its main advantage would seem to lie in the particularly simple analogue equipment needed. Against this must be set: the restriction (not fundamental) to solutions on the boundary only; the restriction to simple contours $C$ closely associated with the $x, y$ coordinate system; the considerable amount of numerical work required for any other than Dirichlet type problems.

*J. G. L. Michel* (Teddington)

## MECHANICS OF PARTICLES AND SYSTEMS

See also A1719, A1866, 1981, 2149, 2202.

2046:

**Fraeijs de Veubeke, B. L'énergie potentielle complémentaire dans les problèmes dynamiques.—Un principe de variation des accélérations.** Ann. Soc. Sci. Bruxelles. Sér. I **73** (1959), 327–344.

Let $T(q_r, \dot{q}_r, t)$ be the kinetic energy of a holonomic system and $V(q_r)$ the potential energy. By introducing $V(h_r)$ in Hamilton's principle, the $h_r$ being $n$ coordinates independent of the $q_r$, and by adding a term $\sum_r f_r(h_r - q_r)$ so that the variation of each of the $n$ multiplicators $f_r$ leads to $h_r - q_r = 0$, the author obtains a principle helpful

in establishing approximative solutions of the equations of motion. As illustrations are treated oscillations of a pendulum and vibrations in continuous medium.

*E. B. Schieldrop* (Oslo)

2047:

**Kalinovič, V. M. On the circulation of an object on the Earth's surface.** Dopovidi Akad. Nauk Ukraïn. RSR **1959**, 837–841. (Ukrainian. Russian and English summaries)

Author's summary: "Formulae are derived for the change in the northern and eastern components of the linear velocity of an object moving along a circle on the Earth's surface. It is shown that the formulae $V \cos \omega t$ and $V \sin \omega t$ which are ordinarily applied to describe this form of motion are inaccurate, being the first approximation of the formulae derived in the article. It is further demonstrated that with large radii of circulation the application of the usually applied formulae may lead to considerable errors in the determination of the northern and eastern components of the linear velocity of the object. The motion taking place in conformity with the formulae $V \cos \omega t$ and $V \sin \omega t$ is not circulation, but a motion along some closed curve inscribed in a spherical zone."

2048:

**Bezborodnikov, M. F. Application of the method of interpolation with corrections to approximate synthesis of mechanisms.** Trudy Inst. Mašinoved. Sem. Teorii Mašin i Mehanizmov. **20** (1959), no. 77, 11–26. (Russian)

2049:

**Nožička, František. Über ein Modell in der klassischen Theorie des Zwei-Körper-Problems.** Apl. Mat. **5** (1960), 1–29. (Czech. Russian and German summaries)

A geometrical model of the path of the motion of one material point in the gravitational field of another material point is presented. Next, some special problems are treated; primarily, the necessary and sufficient conditions for a motion of a material point around a material sphere along an elliptical path which does not intersect the surface of a spherical body.

*R. M. Evan-Iwanowski* (Syracuse, N.Y.)

2050:

**Brdička, Miroslav. Eine Bemerkung zur Abhandlung von František Nožička "Über ein Modell in der klassischen Theorie des Zwei-Körper-Problems".** Apl. Mat. **5** (1960), 30–39. (Czech. Russian and German summaries)

This is a discussion of the paper by F. Nožička [see preceding review]. It is shown that the results obtained geometrically by Nožička, i.e., the elliptic motion of a material point in a gravitational field of a homogeneous sphere or a sphere consisting of concentric spherical shells, can be obtained analytically utilizing the Binet formula.

*R. M. Evan-Iwanowski* (Syracuse, N.Y.)

2051:

**Tokarev, A. I. Equilibration of a solid body rotating about a fixed point.** Trudy Inst. Mašinoved. Sem. Teorii Mašin i Mehanizmov. **19** (1959), no. 73, 39–47. (Russian)

2052:

**Garrido, L. M. Perturbations in classical mechanics.** Proc. Phys. Soc. **76** (1960), 33–35.

Author's summary: "This paper describes a method of evaluating perturbations in classical mechanics which is similar to quantum mechanics time-dependent perturbation theory."

2053:

**Abalakin, V. K. On the motion of a mass-point within the gravitating heterogeneous three-axial ellipsoid.** Byull. Inst. Teoret. Astr. **7**, 327–353 (1959). (Russian. English summary)

From the author's summary: "The present paper deals with the motion of a mass-point within the heterogeneous three-axial ellipsoid, the effects of medium resistance being neglected. The density distribution is assumed to be depending on the polar semiaxis of the ellipsoid, the layers of equal density having homothetical ellipsoidal surfaces as boundaries. By applying the Lindstedt-Lyapounov method the solutions are obtained corresponding to motions of the mass-point in an arbitrarily inclined plane describing either closed or unclosed rosettes within a certain 'middle' sphere. The rosettes are closed when the period of the solution in question is commensurable with $2\pi$. These solutions will be used as generating ones for the construction of a general solution."

*R. M. Evan-Iwanowski* (Syracuse, N.Y.)

2054:

**Quilghini, Demore. Una interpretazione geometrica della distribuzione degli assi principali di inerzia di un sistema materiale.** Boll. Un. Mat. Ital. (3) **14** (1959), 399–404. (English summary)

2055:

**Trombley, E. F. The approximate solution of the equations of motion of an airplane moving in a vertical plane.** J. Aero/Space Sci. **27** (1960), 394–396.

2056:

**Pottsepp, L.; v. Krzywoblocki, M. Z. On the application of the stability theory of differential systems to the stability of a missile. I, II.** Acta Phys. Austriaca **13** (1960), 48–64, 321–338.

Classical results from the theory of stability of motion are applied to a missile possessing rotational symmetry and a plane of mirror symmetry.

*J. K. Hale* (Baltimore, Ma.)

2057:

**Mozniker, R. A. Forced vibrations of mechanical systems in a field of electromagnetic exciters.** Dopovidi Akad. Nauk Ukraïn. RSR **1959**, 1064–1070. (Ukrainian. Russian and English summaries)

Author's summary: "The author shows that linear mechanical systems undergo non-linear vibrations under the action of the magnetic field of electromagnetic exciters. Formulae have been obtained for the determination of the natural frequency of these vibrations, as

well as relationships of disturbing forces in the case of ante-resonance vibrations of equal amplitudes. Theoretical computations have been found to be in agreement with the experimental results."

2058:

**Rosenberg, R. M. Normal modes of nonlinear dual-mode systems.** J. Appl. Mech. **27** (1960), 263–268.

The author considers a non-linear two-degree-of-freedom spring-mass system in which the relation between the spring force $F$ and the corresponding extension $x$ is: $F=\sum_n a_n x^n$, $n=1, 3, 5, \cdots$. A concept of normal modes is defined; knowledge of these modes reduces the coupled system to two uncoupled ones which can be integrated in quadrature. It is shown that the relation between the coordinates of the two masses in a normal mode is a straight line (a) when $F=ax^3$ and (b) for a symmetric system. An iterative method is given for cases in which the modal relation is a curve.

*G. B. Warburton* (Edinburgh)

2059:

**Caughey, T. K. Classical normal modes in damped linear dynamic systems.** J. Appl. Mech. **27** (1960), 269–271.

It is shown that a necessary and sufficient condition for a damped linear system to possess classical normal modes is that the damping matrix be diagonalized by the transformation that uncouples the undamped systems. Rayleigh's solution (that the damping matrix should be a linear combination of the stiffness and inertia matrices) is a special case of the above condition.

*G. B. Warburton* (Edinburgh)

2060:

**Paslay, P. R.; Gurtin, M. E. The vibration response of a linear undamped system resting on a nonlinear spring.** J. Appl. Mech. **27** (1960), 272–274.

2061:

**Grandori Guagenti, Elisa. Solido a struttura giroscopica che rotola lungo una sua circonferenza su di un cilindro rotondo in moto traslatorio.** Ist. Lombardo Accad. Sci. Lett. Rend. A **93** (1959), 423–438.

The author studies the motion of a homogeneous rigid body of revolution firmly connected with a circumference with the same axis, when this circumference rolls without slipping on a vertical circular cylinder which it surrounds, while the cylinder performs a given translatory motion. In particular, the translatory motion of the cylinder is chosen such that the point of contact between the cylinder and the circumference describes a horizontal circle or a helix on the cylinder, while the angle of inclination of the circumference remains constant.

*H. Tornehave* (Copenhagen)

2062:

**Gray, Andrew. ★A treatise on gyrostatics and rotational motion. Theory and applications.** Dover Publications, Inc., New York, 1960. xx + 530 pp. Paperbound: $2.75.

This edition is an unabridged and unaltered publication of the work first published in 1918 [Macmillan, London]. The chapter headings are: 1. Introductory. 2. Dynamical principles. 3. Elementary discussion of gyrostatic action. 4. Systems of coordinates and their relations. Space-cone and body-cone. 5. The simpler theory of tops and gyrostats. 6. Further discussion of the rise and fall of a top when the initial precession is not zero. 7. Gyrostats and various physical applications of gyrostats. 8. Vibrating systems of gyrostats. Suggestions of gyrostatic explanation of properties of matter. 9. The motion of chains of gyrostatic links. Magneto-optic rotation. 10. The earth as a top. Precession and nutation. Gyrostatic theory of the motion of the nodes of the moon's orbit. 11. The free precession of the earth. Further discussion. 12. Calculation of the path of the axis of a top by elliptic integrals. 13. Liquid gyrostat. Miscellaneous investigations. 14. Effects of air friction and pressure. Boomerangs. 15. The spherical pendulum. Motion of a particle on a surface of revolution. 16. Dynamics of a moving frame containing a flywheel. 17. Motion of an unsymmetrical top. 18. The rising of a symmetrical top supported on a horizontal surface. 19. General dynamics of gyrostatic and cyclic systems. 20. Theory of gyrostatic domination. 21. Geometrical representation of the motion of a top. 22. Analogy between a bent rod and the motion of a top. Whirling of shafts, chains, etc. 23. Examples of gyrostatic action and rotational motion.

2063:

**Graham, E. W. A class of optimum trajectory problems in gravitational fields.** J. Aero/Space Sci. **27** (1960), 296–303.

The author considers the problems involving the transfer of a rocket vehicle from one point to another with minimum fuel expenditure. The problems are treated in a nonrotating, but uniformly translating coordinate system. Thus, at time $t=0$ a particle is at the origin traveling with vector velocity $\mathbf{V}_0$. At time $t=T$ it must again be at the origin but traveling with vector velocity $\mathbf{V}_F$. How is this to be accomplished with the minimum $\Delta V$? To answer this question the author follows some sort of a graphical method. Suppose we permit impulses at any finite number of intermediate times, so that the path between two points may consist of many straight line elements. The conclusion is that the optimum path is a straight line with impulsive thrusts applied only at both end points. This graphical reasoning is applied to the force-free, uniform gravitational and the centrally directed gravitational fields. The latter is the most interesting one leading to a diagram of a parameter measuring the importance of the local central force field nonuniformity vs. angle between the vectors. A table of prescriptions of how to apply the method in a particular case closes the paper. With some effort the proposition could be put onto a rigorous analytical base.

*M. Z. v. Krzywoblocki* (E. Lansing, Mich.)

2064:

**Chang, Y. C.; Chang, Chia-hsiang. An investigation on the orbital motion of an artificial satellite.** Acta Astr. Sinica **5** (1957), 196–221. (Chinese. English summary)

Supposing that an artificial satellite has been sent up to a place from which the orbital motion begins under conditions previously proposed, the authors investigate the variations of its orbital elements, as is usual in

celestial mechanics, by taking into account the effects due to both the oblateness of the earth and the atmospheric resistance. The duration of its motion and the time to fall from its orbit to the earth are then deduced in several cases. A certain condition regarding the proposed inclination of its orbit to the equator is also discussed.

In forming the law of atmospheric resistance, the authors make use of two different formulae in comparison to compute the atmospheric density, the one being entirely empirical and the other theoretically deduced from the assumption based on the Rocket Panel atmospheric data that there exists a linear relation between the absolute temperature ($T$) at a point above a height of 100 km. and its geocentric distance ($r$) as follows: $T = 6\times10^{-5}r - 3.868\times10^{4}$ in c.g.s. units. *Zee Shen* (Taipeh)

2065:

**Irimiciuc, N. La méthode de la réduction des masses dans l'étude du mouvement général du solide dont la masse est variable.** Bul. Inst. Politehn. Iaşi (N.S.) **4 (8)** (1958), no. 3–4, 89–92. (Romanian. Russian and French summaries)

Si stabiliscono le equazioni di Lagrange per un sistema di punti di masse variabili equivalente dal punto di vista dinamico ad un solido. Si dimostra inoltre, conservandosi le ipotesi che vi hanno condotto V. S. Novoselov [Vestnik Leningrad. Univ. **12** (1957), no. 1, 130–140, 210; MR **19**, 695] alla deduzione delle equazioni di Lagrange per un solido di massa variabile (le equazioni (1) della nota citata), l'equivalenza del sistema di equazioni ottenuta con quella stabilità dal Novoselov (eq. (1) della nota citata). *D. Mangeron* (Iaşi)

## STATISTICAL THERMODYNAMICS AND MECHANICS

See also 2020, 2391.

2066:

**Morozova, T. V. Asymptotic inequalities applicable to problems of statistical mechanics.** Ukrain. Mat. Ž. **11** (1959), 321–328. (Russian)

2067:

**Levine, S.; Bell, G. M. Theory of a modified Poisson-Boltzmann equation. I. The volume effect of hydrated ions.** J. Phys. Chem. **64** (1960), 1188–1195.

Authors' summary: "By applying Kirkwood's work, which is based on classical statistical mechanics, the general form of the correction to the Poisson-Boltzmann equation due to the short-range forces between the ions is obtained in the case of the diffuse layer of a single, charged colloidal particle immersed in an aqueous electrolyte. The correction can be expanded in powers of the electrolyte concentration and terms up to the square of the concentration are determined. The general expansion is applied to the particular case of a plate-like particle immersed in a 1-1 electrolyte in which it is assumed that the hydration shells of the positive and negative ions are impenetrable and have the same diameter. This is compared with the so-called method of local thermodynamic balance, which has been used by a number of authors. Here, however, the volume fraction statistics of Flory has been assumed, in preference to the various interpretations of molar fraction statistics considered by these authors. It is shown that, provided the distance from the plate is greater than two or three molecular diameters, this method will give the correct form of the term in the above expansion which is linear in the electrolyte concentration. However, it fails to reproduce correctly the next (second-order) term, proportional to the square of the concentration, the principal reason being that no account is taken of the variation of the mean electrostatic potential over a distance of the diameter of the hydration shell. It is also demonstrated that the form of the volume correction to the Poisson-Boltzmann equation changes in the immediate vicinity of the colloidal plate."

2068:

**Levelt, J. M. H.; Hurst, R. P. Quantum mechanical cell model of the liquid state. I.** J. Chem. Phys. **32** (1960), 96–104.

This paper represents a study of the quantum-mechanical generalization of the primitive cell model for liquids due to Lennard-Jones and Devonshire. It is well known that this model, in the classical theory, gives a rather good account of some, but not all, of the thermodynamic properties of liquids. It is based on the following unrealistic assumptions: (i) the number of nearest neighbours of a given atom is the same in the liquid as in the crystal; (ii) to determine the potential energy of an atom in its cell, neighbouring atoms may be placed in the centres of their cells; (iii) empty or multiply occupied cells and correlations between configurations in neighbouring cells are disregarded. To these, the authors have added another: (iv) the atomic density is zero on the cell boundary.

Having admitted the approximations resulting from these assumptions, the authors have solved the resulting quantum-mechanical problem with great numerical accuracy, and computed the internal energy and the specific heats for liquid hydrogen and deuterium. As expected, the results deviate considerably from those of the classical theory at very low temperatures, but approximate to the classical results at reduced temperatures greater than 3. *H. S. Green* (Adelaide)

2069:

**Broyles, A. A. Radial distribution functions from the Born-Green integral equation.** J. Chem. Phys. **33** (1960), 456–458.

Author's summary: "Comparisons are made between solutions to the Born-Green integral equation and radial distribution functions obtained by the Monte Carlo method by Wood and Parker for the Lennard-Jones potential. It is observed that multiplying the particle separation distance in the Born-Green case by a constant factor improves the agreement for loops beyond the first."

2070:

**Linder, Bruno. Continuum-model treatment of long-range intermolecular forces. I. Pure substances.** J. Chem. Phys. **33** (1960), 668–675.

Author's summary: "A theory is presented whereby the long-range intermolecular forces, including the London dispersion forces, of pure nonelectrolytes may be

calculated from optical and dielectric data. The method is based on the continuum-model approach, where one molecule is treated explicitly while the others are replaced by a medium of uniform dielectric. The classical and quantum-mechanical oscillators are used as working models and expressions are derived for computing the cohesive energy appropriate for both types of oscillators. The potential energy based on the quantum-mechanical oscillator is calculated for a number of liquids and is shown to be in fair agreement with the experimental energy of vaporization."

2071:

**Steele, William A.; Ross, Marvin. Distribution functions of a fluid in an external potential field: application to physical adsorption.** J. Chem. Phys. **33** (1960), 464–470.

Authors' summary: "The thermodynamic properties of a fluid in an external potential field are expressed in terms of the spatial distribution functions appropriate to the fluid. Several methods of computing these distribution functions are given: integro-differential equations analogous to the Born-Green-Yvon equations; integral equations analogous to the Kirkwood-Salsburg equations; and two approximate methods of computation based on a perturbation approach. The applicability of these methods of calculation to various aspects of the problem of physical adsorption is discussed."

2072:

**Watson, Kenneth M. Use of the Boltzmann equation for the study of ionized gases of low density. I.** Phys. Rev. (2) **102** (1956), 12–19.

Author's summary: "The Boltzmann equation is studied for the case of a low-density ionized gas in an externally applied electromagnetic field. Particle-particle collisions are neglected, but long-range collective interactions are included. In part I the static problem is treated in detail. For this case the Boltzmann equation is solved using individual-particle orbits—an approach which emphasizes the physical basis of the solution."

*M. L. Goldberger* (Princeton, N.J.)

2073:

**Brueckner, K. A.; Watson, K. M. Use of the Boltzmann equation for the study of ionized gases of low density. II.** Phys. Rev. (2) **102** (1956), 19–27.

Authors' summary: "The Boltzmann equation for ionized gases of low density in an external magnetic field is used to obtain approximate solutions in the nonstatic case. The Boltzmann and Maxwell equations are linearized by assuming small deviations from a static solution. It is shown that in the limit of a strong magnetic field ($\eta \ll 1$, as defined in the text), the motion transverse to the magnetic field is described by the conventional hydrodynamic equations. The variation along field lines is described by a one-dimensional (i.e., one space dimension and one velocity dimension) Boltzmann equation. Several applications are given, including an analysis of the Kruskal-Schwarzschild gravitational instability of a plasma."

*M. L. Goldberger* (Princeton, N.J.)

2074:

**Manley, O. P. A method of evaluating diffusion coefficients in crystals.** Phys. and Chem. Solids **13** (1960), 244–250.

Author's summary: "The mechanism of diffusion proposed by Rice is reconsidered with the aid of an extension of Kac's theorem. The analysis yields an activation energy which, as predicted by Zener, is simply related to the minimal local deformation energy. It is also found that in contrast with Rice's treatment, the activation energy may be calculated directly from the atomic force constants without resorting to normal mode analysis."

2075:

**Neuberger, Jacob; Hatcher, Robert D. Frequency spectrum for a two-dimensional lattice.** J. Chem. Phys. **33** (1960), 265–269.

Authors' summary: "The methods of lattice dynamics are applied to a study of the frequency distribution function $f(\nu)$ of a two-dimensional model of a monatomic crystal. It is shown that if the interaction of the atoms of the lattice is of short range such that next-nearest neighbor interactions are much weaker than nearest neighbor interactions, the function $f(\nu)$ can be expressed in terms of elliptic integrals of the first kind. These functions are calculated for a particular choice of the ratio of the interaction constants which had previously been discussed in the literature by approximate numerical methods. This gives some evidence that the numerical scheme for obtaining $f(\nu)$, based on a comparatively small spanning of wave-vector space which is used extensively in more complicated crystal models, is adequate provided that $\nu$ is a fairly smooth function of the wave vector."

2076:

**Bellman, Richard; Kalaba, Robert; Wing, G. Milton. Invariant imbedding and neutron transport in a rod of changing length.** Proc. Nat. Acad. Sci. U.S.A. **46** (1960), 128–130.

The Chandrasekhar-Ambarzumbian method of invariant imbedding [see the article by the same authors in J. Math. Mech. **7** (1958), 149–162; MR **20** #2046] is applied to a hypothetical homogeneous rod of variable length, in which monoenergetic neutrons move longitudinally, each collision producing two fission neutrons.

*G. Birkhoff* (Cambridge, Mass.)

2077:

**Snider, R. F.; Curtiss, C. F. Kinetic theory of moderately dense gases.** Phys. Fluids **1** (1958), 122–138.

This important contribution to the kinetic theory of gases shows in detail how to compute the coefficients of viscosity and thermal conduction for imperfect gases, given the intermolecular potential energy. Though it only takes account of binary encounters, other approximations implicit in the original formulation of Boltzmann's equation are avoided; the results are therefore exact to terms linear in the density, and allow terms proportional to the square of the density to be estimated. The analysis is too complicated to be described in detail.

*H. S. Green* (Adelaide)

2078:

**Rice, Stuart A.; Kirkwood, John G.; Ross, John; Zwanzig, Robert W. Statistical mechanical theory of transport processes. XII. Dense rigid sphere fluids.** J. Chem. Phys. **31** (1959), 575–583.

A modified Maxwell-Boltzmann integro-differential equation for the distribution function in $\mu$ space (coarse-grained in time) for a gas of hard spheres at moderate densities is derived from the Liouville equation subject to certain plausible but unverified assumptions: (1) The dynamics of the system is determined by binary collisions only. (2) The non-equilibrium pair correlation function is taken to be independent of momenta and is approximated by the equilibrium pair correlation function. (3) The characteristic time interval chosen is so short that the changes in intermolecular separation during the time smoothing interval are negligible. The resulting theory attempts to correct for at least two inadequacies of the usual low density Boltzmann transport theory, the deviation of the pair correlation function from unity and the inclusion of collisional transfer in the evaluation of the flux vectors. The modified derived transport equation differs from the Boltzmann equation in that the coordinates of the colliding molecules are kept distinct in the collisional change of $J$, with time and that this term is multiplied by the equilibrium pair-correlation function on contact of two hard spheres. The differences between the newly derived equations and those of Enskog and Snider and Curtiss [#2077 above] are noted. The computed viscosity of a dense hard sphere fluid agrees, while the thermal conductivity differs, from that first computed by Enskog.

*H. L. Frisch* (Murray Hill, N.J.)

2079:

**Helfand, Eugene; Rice, Stuart A. Principle of corresponding states for transport properties.** J. Chem. Phys. **32** (1960), 1642–1644.

Authors' summary: "The principle of corresponding states can be demonstrated by use of the autocorrelation function expressions for the transport properties and the assumption that the intermolecular potential has the form $u = \varepsilon u^*(r/\sigma)$. The result follows from the fact that both the canonical ensemble distribution function and the solution of the mechanical equations of motion may be written in reduced variables. One finds that $\eta^* = \eta\sigma^3/m^{1/2}\varepsilon^{1/2}$, $\kappa^* = \kappa k\varepsilon^{1/2}/m^{1/2}\sigma^2$, and $D^* = Dm^{1/2}/\varepsilon^{1/2}\sigma$ are universal functions of $T^* = Tk/\varepsilon$, $P^* = P\sigma^3/\varepsilon$, and in the quantum mechanical case $\hbar^* = \hbar/\sigma m^{1/2}\varepsilon^{1/2}$."

2080:

**Shuler, Kurt E. Relaxation of an isolated ensemble of harmonic oscillators.** J. Chem. Phys. **32** (1960), 1692–1697.

Author's summary: "The collisional relaxation of an isolated ensemble of harmonic oscillators (at constant volume and energy) from initial nonequilibrium distributions is discussed in this paper. The 'transport equation' for the relaxation process is derived and it is shown that it can be linearized even though the relaxation takes place via binary oscillator collisions. The final, stationary distribution is found to be a Boltzmann one with a temperature uniquely defined by the mean energy of the ensemble. The Boltzmann $H$ function is obtained for this system of relaxing oscillators and it is shown that $dH/dt < 0$ for all $t$. The time rate of change of the mean-square deviation of the energy during the relaxation process is computed and is shown to be closely related to the time variation of the mean energy in the relaxation of an ensemble of harmonic oscillators in contact with a thermal reservoir."

2081:

**Khinchin, A. Y. [Hinčin, A. Ya.] ★Mathematical foundations of quantum statistics.** Translation from the first (1951) Russian ed., edited by Irwin Shapiro. Graylock Press, Albany, N.Y., 1960. xi+232 pp. $10.00.

This book includes an English translation of (*) *Matematičeskie osnovaniya kvantovoĭ statistiki* [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1951; MR **13**, 894] as made by E. J. Kelly, Jr., M. D. Friedman, W. H. Furry and A. H. Halperin. Prior to being translated from Russian into English (*) was translated from Russian into German [*Mathematische Grundlagen der Quantenstatistik*, Akademie-Verlag, Berlin, 1956; MR **18**, 443]. In addition to (*) the book contains the translation of two articles by the author [(i) Trudy Mat. Inst. Steklov **38** (1951), 345–365; and (ii) *Pamyati Aleksandra Aleksandroviča Andronova*, pp. 541–574, Izdat. Akad. Nauk SSSR, Moscow, 1955; MR **13**, 895; **17**, 567]. The book, written by an eminent Russian tychist (i.e., a probabilist, or one who deals with measure-theoretical analysis), is a natural extension of an earlier work by the author [*Mathematical foundations of statistical mechanics*, Dover, New York, 1949; MR **10**, 666], the measure-theoretical substance of which has been briefly summarized by Truesdell [Science **127** (1958), 729–739; MR **20** #1437; pp. 737, 738].

The spirit of the presentation of the text has certain pedagogical motivations. In addition the spirit represents a compromise between the wishes of mathematicians who want to become acquainted with some applications of analysis to physics and physicists interested in a mathematical foundation for their science. The first two chapters of the book are preliminary in nature. Chapter 1 treats of local limit theorems for sums of identically distributed random variables that assume non-negative integral values. Complete proofs of the local limit theorems are given for the one- and two-dimensional cases [B. V. Gnedenko and A. N. Kolmogorov, *Limit distributions for sums of independent random variables*, Addison-Wesley, Cambridge, Mass., 1954; MR **16**, 52]. Chapter 2 deals with basic concepts and terminology of quantum mechanics. If it was not a matter of learning terminology a respectable physicist could omit chapter 2. A professional mathematician should, but probably will not spend much time with chapter 2, even though this chapter does not contain "physical experiments". Chapter 3 considers general principles of quantum statistics. Here the author studies, among other things, the distribution laws of various physical quantities for a system that has a given total energy and shows that the problem of establishing the suitability of microcanonical averages can be reduced to that of estimating the microcanonical dispersions of the corresponding physical quantities. Chapter 4 treats of the foundations of the statistics of photons. Here the author gives a specific structure to quantum statistics so as to treat particles with zero rest mass. Chapter 5 extends chapter 4 to particles with non-zero rest mass. In chapter 6 the author defines the concept of entropy and by

statistical methods deductively arrives at the second law of thermodynamics. Of course, the first law of thermodynamics can be arrived at deductively, but without statistical methods, as a consequence of Schrödinger's equation. The supplements to the book contain material of indirect interest to the development of the main trend of the mathematical theory.

The reviewer places the author's physico-mathematical treatise on the level of quality of J. von Neumann's book [*Mathematical foundations of quantum mechanics*, Princeton Univ., Princeton, N.J., 1955; MR **16**, 654] or I. E. Segal's notes [*A mathematical approach to elementary particles and their fields*, Department of Mathematics, University of Chicago, 1955].

*A. A. Mullin* (Urbana, Ill.)

2082:

**Jancel, Raymond. Propriétés des observables macroscopiques et théorie ergodique quantique.** C. R. Acad. Sci. Paris **250** (1960), 671–673.

It is shown that the dispersion of a macroscopic observable tends to zero in a microcanonical ensemble as the number of degrees of freedom tends to infinity. The relation between this and ergodic behaviour is discussed.

*D. ter Haar* (Oxford)

2083:

**Jancel, Raymond. Comparaison entre les aspects classique et quantique de la théorie ergodique.** C. R. Acad. Sci. Paris **250** (1960), 2152–2154.

The equivalence of the quantum mechanical ergodic theorem [as developed by the author: see preceding review] and the classical one (as stated by Khinchine) is pointed out, as well as the difference between the two which consists in the fact that the quantum mechanical theory involves the concept of macroscopic observables. If one considers a system of independent, identical particles, this last difference does not occur.

*D. ter Haar* (Oxford)

2084:

**Yvon, J. Mécanique statistique quantique. Opérateurs densités et grandeurs thermodynamiques.** J. Phys. Radium **21** (1960), 569–574. (English summary)

Author's summary: "The simple and double density operators, as functions of temperature and activity, are computed for a system in equilibrium. The scattering operator allows, in some cases, separating out the thermodynamical variables. In particular, these variables do not appear in the expression of the double density operator, as a function of the simple density operator; which explains the fact that this expression is still valid out of equilibrium."

2085:

**Mandelbrot, Benoît. Les ensembles grand-canoniques de Gibbs; justification de leur unicité, basée sur la divisibilité infinie de leur énergie aléatoire.** C. R. Acad. Sci. Paris **249** (1959), 1464–1466.

2086:

**Schafroth, M. R. On the foundations of equilibrium statistical mechanics.** Helv. Phys. Acta **32** (1959), 349–356.

It is emphasized that the quantum mechanical ensembles can be generated operationally through suitably chosen idealized experiments. One can then define thermal equilibrium as that state where no change in the ensemble is any longer possible. This entails that the ergodic theorem belongs to statistical mechanics of non-equilibrium states; as there is no satisfactory development of non-equilibrium statistical mechanics one should not be surprised at the difficulties ergodic theory has run into.

*D. ter Haar* (Oxford)

2087:

**Bernard, William; Callen, Herbert B. Irreversible thermodynamics of nonlinear processes and noise in driven systems.** Rev. Mod. Phys. **31** (1959), 1017–1044.

This review treats the general problem of the response of thermodynamic variables $Q_i$ and their fluctuations to externally applied, time-dependent forces $F_i$. The development is quantum mechanical, although corresponding classical results are given in a number of cases. The basic procedure used is to expand the operator $Q_i(t)$ into a power series in $F_1$, $F_2$, etc. and then to take the desired ensemble averages $\langle Q_i(t)\rangle$, $\langle [Q_i(t), Q_j(t+\tau)]_+\rangle$, etc. In this way $\langle Q_i(t)\rangle^{(1)}$, the contribution to $\langle Q_i(t)\rangle$ which is of first order in the applied forces, can be related to the averages $\langle [Q_i(t), Q_j(t+\tau)]_-\rangle^{(0)}$, evaluated for an equilibrium ensemble in the absence of applied forces. Similar expressions are given for $\langle Q_i(t)\rangle^{(2)}$, $\langle [Q_i(t), Q_j(t+\tau)]_+\rangle^{(1)}$, and $\langle [Q_i(t), Q_j(t+\tau)]_-\rangle^{(2)}$ in terms of equilibrium averages of more complex quantities, involving triple and quadruple products of $Q$'s. The special cases of step-function and $\delta$-function forces are treated in detail; the first order response in the latter case is the so-called after-effect function, and the relationship of this to the equilibrium average $\langle [Q_i(t), Q_j(t+\tau)]_+\rangle^{(0)}$ is obtained both in terms of the functions themselves and in terms of their Fourier transforms. Most of the remainder of the paper is devoted to consideration of the higher order terms in $\langle Q_i(t)\rangle$ and $\langle [Q_i(t), Q_j(t+\tau)]_+\rangle$. The last two sections deal with the distribution function $W_1(Q, t)$, the probability that the $Q$'s have a given set of values at time $t$, and the analogously defined joint distribution $W_2(Q, t; Q', t')$.

*S. Prager* (Minneapolis, Minn.)

2088:

**George, Cl. Phénomènes irréversibles dus à la substitution isotopique dans les cristaux.** Acad. Roy. Belg. Bull. Cl. Sci. (5) **45** (1959), 239–250. (English summary)

The angle-action variable method of Prigogine and coworkers is applied to the dynamics of a crystal whose atoms interact according to Hookean potentials, but in which equivalently situated atoms may have slightly different masses. The relaxation of a single vibration mode interacting with a set of modes in thermal equilibrium is discussed.

*S. Prager* (Minneapolis, Minn.)

2089:

**Thomsen, John S. Thermodynamics of an irreversible quasi-static process.** Amer. J. Phys. **28** (1960), 119–122.

The author considers the case of a gas confined by a piston whose motion is subject to sliding friction, so that the force $F_+$ required to move it forwards at an infinitesimal rate differs by a finite amount from the force $F_-$

required to move it backwards. The manner in which the force $F_r$ exerted by the gas upon the piston varies with temperature $T$ at a fixed piston displacement $X$ is given by the combined first and second laws of thermodynamics, although the absolute value of $F_r$ is not. The latter must, however, be so chosen that $F_r$ lies between $F_+$ and $F_-$ for all values of $X$ and $T$.

*S. Prager* (Minneapolis, Minn.)

2090:

**van Kampen, N. G. The definition of entropy in non-equilibrium states.** Physica **25** (1959), 1294–1302.

Author's summary: "Several different expressions are used in statistical mechanics for the entropy. They are here evaluated explicitly for non-equilibrium states. The result consists of three terms: (i) the equilibrium entropy; (ii) the familiar quadratic term measuring the departure from equilibrium; (iii) a term referring to the fluctuations. Only the last term differs for the several expressions for the entropy. However, it is much smaller than the second term when the departure from equilibrium is large compared to the size of the fluctuations."

*W. Byers Brown* (Manchester)

2091:

**Magalinskij, V. B.; Terletskij, Ja. P. On the statistical theory of nonequilibrium processes.** Ann. Physik (7) **5** (1960), 296–307.

A general method is worked out on the basis of principles of Gibbs' statistical mechanics which allows one to find stationary probabilities and transition probabilities for physical quantities provided either the behaviour of their mean values or the general form of corresponding equations of motion (Langevin equations) is known.

The proposed method is free from ordinary restrictions of the theory of fluctuations and Brownian motion, such as modest fluctuations, linearity of phenomenological equations, Markov's character of the random process, and is applicable also to postacting systems.

The method is applied to some physical systems.

For the system described by a nonlinear equation of motion the general coordinate-velocity equation of motion of the transition probability density is obtained. This equation which coincides with the well-known Einstein-Fokker-diffusion equation is obtained for the electrical circuit consisting of capacity and nonlinear resistance. Its general solution is found. *J. Ross* (Providence, R.I.)

## ELASTICITY, PLASTICITY

See also 1981.

2092:

**Rüdiger, D. Eine Verallgemeinerung des Prinzips vom Minimum der potentiellen Energie zweidimensionaler elastischer Kontinua.** Ing.-Arch. **29** (1960), 115–124.

In the general boundary value problem of elastostatics the kinematic boundary conditions concern surface displacements and the static boundary conditions involve surface tractions. The principle of minimum potential energy singles out the displacement field of the solution from all displacement fields that possess the necessary continuity and differentiability properties and satisfy the kinematic boundary conditions. An analogous statement can be made concerning the principle of minimum complementary energy, which characterizes the stress field of the solution. In both cases, the solution can be shown to furnish an absolute minimum. E. Reissner [Proc. Symp. Appl. Math., Vol. VIII, pp. 1–6, Amer. Math. Soc., Providence, R.I., 1958; MR **20** #2903] has established a variational principle that characterizes both the stress and the displacement fields of the solution, furnishing the differential equations of elastostatics as Euler equations and the boundary conditions as natural boundary conditions. The variational principle of the present problem takes an intermediate place between the classical extremum principles and the variational principle of Reissner. It singles out the displacement field of the solution from all displacement fields with the necessary continuity and differentiability properties, regardless of whether they fulfil the kinematic boundary conditions. (Unfortunately, the principle is stated in a form that does not clearly exhibit its structural relation to the principle of minimum potential energy. It can be cast into an alternative form, in which the minimand is obtained from the potential energy by adding a surface integral extended over that part of the boundary where surface displacements are prescribed. The integrand is the scalar product of the surface traction computed by Hooke's law from the considered displacement field with the difference between the given and the considered surface displacement.) The application to the elastostatic boundary value problem is discussed, Saint Venant's torsion problem being treated as an example. *W. Prager* (Providence, R.I.)

2093:

**Hetényi, M. A method of solution for the elastic quarter-plane.** J. Appl. Mech. **27** (1960), 289–296.

This paper contains an iterative scheme for the determination (within classical two-dimensional elasticity theory) of the stresses produced in an elastic quarter-plane ($0 \leq x < \infty$, $0 \leq y < \infty$) by prescribed surface tractions. Assuming, without loss in generality, the loading to be confined to the edge $x = 0$, the author extends the given domain to the half-plane $0 \leq x < \infty$, $-\infty < y < \infty$ and symmetrizes the loading with respect to the axis $y = 0$. He then makes use of Flamant's well-known solution to the half-plane problem thus encountered, noting that the latter meets the boundary conditions appropriate to the original problem along $x = 0$, but gives rise to unwanted residual tractions on the edge $y = 0$. These tractions, in turn, are removed by a second application of Flamant's solution, following a symmetrization of the residual problem about the axis $x = 0$, which leads to a violation of the boundary conditions initially posed for $x = 0$. The foregoing iteration process is continued in an obvious manner and yields a solution to the quarter-plane problem in the form of an infinite series of definite integrals. Numerical results, obtained with the aid of a digital computer, are presented for several specific loading cases. The author shows that both the resultant forces and the maximum values of the successive residual tractions tend to zero as the number of iterations is increased indefinitely; he concludes that his successive approximations converge toward the exact solution of the original problem.

An exact solution to the problem considered may alternatively be obtained in closed integral form, through

an adaptation of available solutions for the elastic wedge, which are based on the use of the Mellin-transform. The chief merit of the method proposed in the present paper lies in the fact that it admits an extension to the corresponding three-dimensional problem for the quarter-space. *E. Sternberg* (Providence, R.I.)

2094:

**Heimke, Günther. Exakte Bestimmung des Spannungszustandes bei unstetiger Belastung.** Wiss. Z. Hochsch. Verkehrswes. Dresden **5** (1957), no. 1, 5–11. (Russian, English and French summaries)

From the author's summary: "For a case of discontinuous load, chosen in a way to permit specializations and generalizations into practical cases, the boundary values of the Airy function are derived. The stress function itself is obtained from the solution of the biharmonic boundary value problem; in connection with this, the method of integration employed is shortly described. From this solution, the stress functions for two loadings can be obtained, the determination of which is normally carried out by the method of inversion.

"The superposition of several states of stress gives finally that state of stress (except for the sign) which for an 'impeller wheel' (rotor of an idealized turbine) is superimposed on the state of stress of the rotating circular disk." *R. M. Evan-Iwanowski* (Syracuse, N.Y.)

2095:

**Ling, Chih-Bing; Tsai, Chen-Peng. Stresses in a slab having a spherical cavity under circular bending.** J. Appl. Mech. **27** (1960), 278–282.

This is one of a series of investigations on the effect of a spherical cavity in a solid of revolution when the solid is acted upon by a given stress system symmetrical to the axis of revolution. Three problems of this nature have been investigated previously by the first author in a succession of papers [Quart. Appl. Math. **10** (1952), 149–156; **13** (1956), 381–391; J. Appl. Mech. **26** (1959), 235–240; MR **13**, 886; **19**, 902; **21** #3145].

This paper deals with an infinite slab of thickness $2a$, having a spherical cavity of radius $\lambda a$ located symmetrically between the two plane surfaces, and under the action of a circular bending moment of $-M$ per unit length around its edge at infinity.

*K. M. Morris* (Cardiff)

2096:

**Sneddon, I. N. A note on the axially symmetrical punch problem.** Mathematika **6** (1959), 118–119.

Segedin [Mathematika **4** (1957), 156–161; MR **20** #496] reduces the axially symmetric punch problem for the elastic half-space to the determination of the solution of an integral equation of the type

$$f(r) = (2\pi^{-1}) \int_0^r K(\xi) \sin^{-1}(\xi/r) d\xi + \int_r^a K(\xi) d\xi, \quad 0 < r < a,$$

where $f(r)$ is known and $K(\xi)$ is sought. In this note the author gives a simple solution to the integral equation.

*L. E. Payne* (Newcastle-upon-Tyne)

2097:

**Reichel, Alex. Distribution of stress in the neighbourhood of a wedge indenter.** J. Proc. Roy. Soc. New South Wales **93** (1959), 69–79.

A semi-infinite elastic medium is indented in plane strain by a rigid wedge bounded by the planes $x = \pm l$ for $y \geq \varepsilon l$ and by $y = \varepsilon|x|$ for $|x| \leq l$, $\varepsilon$ being small. The lower faces of the wedge may or may not be in complete contact with the medium. The methods of Muskhelishvili are used to find the components of stress in the medium in both cases. *D. R. Bland* (Manchester)

2098:

**Černin, K. E. Solution of an axisymmetric problem by the straight-line method.** Trudy Mat. Inst. Steklov. **53** (1959), 302–306. (Russian)

2099:

**Kobec, L. G. Fundamental equations of the stress and deformation state of elastic open thin-walled rods with large angle of twist.** Trudy Har'kov. Inž.-Stroit. Inst. Meh. **1** (1957), no. 5, 99–110. (Russian)

2100:

**Solyanik-Krassa, K. V. Elastic equilibrium of bodies of revolution.** Inžen. Sb. **26** (1958), 113–136. (Russian)

General forms of solution of the equations of elasticity in cylindrical coordinates $r$, $\varphi$, $z$ are given as trigonometric series in the coordinate $\varphi$. Application is made to the bending of circular cylinders by forces applied to the curved surface parallel to the axis of the cylinder.

*R. C. T. Smith* (Armidale)

2101:

**Iwinski, T. ★Theory of beams: The application of the Laplace transformation method to engineering problems.** Translated from the Polish by E. P. Bernat. Pergamon Press, New York-London-Paris-Los Angeles, 1958. 85 pp. $3.50.

The book is of interest for an engineer employed in structural mechanics. It could be also used as a reference or text book for an intermediate course in the theory of strength of materials. It covers a variety of problems of beams on two supports, beams with rigid and elastic supports, and of beams of variable rigidity. The book is valuable in the sense that it introduces a fresh mathematical approach to a technical field.

*R. M. Evan-Iwanowski* (Syracuse, N.Y.)

2102:

**Savin, G. M.; Goroško, O. O. Parameters of a naturally twisted thread.** Dopovidi Akad. Nauk Ukraïn. RSR **1959**, 828–832. (Ukrainian. Russian and English summaries)

Authors' summary: "To describe the property of real ropes to untwist on longitudinal tension, a model of such ropes—a naturally twisted thread—is introduced. The principal parameters of the model are: $EF$ and $B$—longitudinal and torsional rigidity of the thread respectively—and $k$, the coefficient of untwisting."

2103:
**Yoshitake, Hiroyuki. Stress distribution in the neighbourhood of the loading point of girder subjected to a concentrated load.** Bull. JSME **3** (1960), 71–76.

2104:
**Roth, W. Die tordierte, einfach gekrümmte Welle mit konstanter Krümmung.** Ing.-Arch. **27** (1960), 326–349.

2105:
**Langefors, B. Practical aspects of structural analysis.** Z. Flugwiss. **8** (1960), 161–168. (German and French summaries)

2106:
**Argyris, J. H.; Kelsey, S. On the matrix theory of structures: conclusion of discussion.** Z. Flugwiss. **8** (1960), 169–172.

2107:
**de C. Henderson, J. C. Topological aspects of structural linear analysis. Improving the conditioning of the equations of compatibility of a multi-member skeletal structure by use of the knowledge of topology.** Aircraft Engrg. **32** (1960), 137–141.

Author's summary: "The selection of a system of cut releases which would render a continuous skeletal structure statically determinate is governed principally by the topological and geometric characteristics of the structure, and to a lesser extent by its material properties. Only the topological aspects will be considered here, the other factors being purposely excluded. It is shown how the knowledge of the topology of the structure can be exploited to improve the conditioning of the associated equations of compatibility. Owing to difficulties inherent in the design of any automatic procedure for the reduction to a canonical form, a method of condensation has been evolved for complex multi-member structures. In this way the large scale features of the structure are taken into account without neglecting the effect of local characteristics."

2108:
**Babuška, Ivo; Fiedler, Miroslav. Über Systeme linearer Gleichungen vom Typ der Rahmentragwerke.** Apl. Mat. **4** (1959), 441–455. (Czech and Russian summaries)

In dieser Arbeit wird die notwendige und hinreichende Bedingung dafür angegeben, daß zu einem gegebenen System linearer Gleichungen ein Rahmentragwerk mit unverschiebbaren Rahmenknoten existiert, dessen Deformationsgleichungssystem das gegebene System ist.

*W. Zerna* (Hannover)

2109:
**Pliev, S. B. Equilibrium of an elastic cylinder of finite dimensions.** Izv. Akad. Nauk Azerbaĭdžan. SSR. Ser. Fiz.-Teh. Him. Nauk **1958**, no. 1, 25–36. (Russian. Azerbaijani summary)

2110:
**Demonsablon, Philippe. Le calcul des piles déformables avec appuis en caoutchouc.** Ann. Ponts. Chaussées **130** (1960), 495–562. (English summary)

2111:
**Hiba, Miodrag; Cederwall, Krister. Flambement élastique d'une barre en bois lamellée et clouée avec le module de déplacement du moyen de liaison constant $K$.** Chalmers Tekn. Högsk. Handl. No. 230 (1960), 22 pp.

2112:
**Lyubimov, V. M. Approximate solution of a problem of an elastic ring sector for certain particular load conditions.** Inžen. Sb. **26** (1958), 137–147. (Russian)

2113:
**Levin, Eugene. Elastic equilibrium of a plate with a reinforced elliptical hole.** J. Appl. Mech. **27** (1960), 283–288.

Equations are presented for a general solution of the stress in a plate with a reinforced elliptical hole. The reinforcement is of constant thickness and has an outer boundary confocal with the hole. The method of Muskhelishvili is used to achieve the solution. An explicit solution of the equations is not obtained by the author; however, he indicates an iteration procedure which is well suited to solution by numerical techniques. For the special case of a circular hole an explicit solution is obtained which agrees with earlier results. For the circular hole the author presents numerical results showing that the greatest stress reduction for a given weight of reinforcing material occurs when the reinforcement is concentrated close to the edge of the hole.

*S. Levy* (Schenectady, N.Y.)

2114:
**Yu, Yi-Yuan. A new theory of elastic sandwich plates—one-dimensional case.** J. Appl. Mech. **26** (1959), 415–421.

The equations of motion of a symmetrical sandwich of three Timoshenko beams are derived and the dispersion-equation for flexural waves is deduced.

*R. D. Mindlin* (New York)

2115:
**Kurata, M. Bending of simply supported rectangular plates with clamped portions along arbitrary sections of the edges.** Ing.-Arch. **27** (1960), 385–416.

In the introduction to this paper the author states that problems on the bending of rectangular thin plates have been worked out for almost all combinations of edge conditions but that the particular problems stipulated in the title have been left comparatively untouched. In order to fill this gap this paper gives a method of solving such problems in as general a form as possible. The solution is obtained by calculating the slope along the clamped portions of the plate under the action of the given load only. These slopes are then cancelled by the applications of moments to the plate along these same portions. Numerical results are given for the following square plates subjected to a uniformly distributed load: (i) a

clamped portion in the middle of one edge; (ii) a clamped portion the same as in (i) on every edge; (iii) a clamped portion at every corner. Comparison is made with experimental results. *R. M. Morris* (Cardiff)

2116:

**Morgenstern, Dietrich. Herleitung der Plattentheorie aus der dreidimensionalen Elastizitätstheorie.** Arch. Rational Mech. Anal. **4**, 145–152 (1959).

Es wird gezeigt, wie sich aus der dreidimensionalen Elastizitätstheorie die Gleichungen der Kirchhoffschen Plattentheorie bei verschwindender Plattendicke ableiten. Es zeigt sich, daß die sonst benutzte Bernoullische Hypothese bei abnehmender Plattendicke mehr und mehr gilt. Beim Vergleich mit früheren Betrachtungen über die Theorie dicker Platten wird festgestellt, daß diese vom mathematischen Standpunkt aus gesehen nicht befriedigen, da an irgendeiner Stelle das "St. Venantsche" Prinzip in einer noch nicht bewiesenen Weise verwendet wird.

Das Beweisverfahren gründet sich auf die Minimalsätze der Elastizitätstheorie. Es werden sowohl das Prinzip vom Minimum der Formänderungsarbeit als auch das Prinzip vom Minimum der Ergänzungsarbeit verwendet. *W. Zerna* (Hannover)

2117:

**Salerno, V. L.; Goldberg, M. A. Effect of shear deformations on the bending of rectangular plates.** J. Appl. Mech. **27** (1960), 54–58.

The general solution of the reviewer [same J. **12** (1945), A-69–A-77; MR **7**, 42] is applied to problems of a rectangular plate with uniformly distributed load and with all four edges simply supported or two opposite edges simply supported and two opposite edges free. Numerical comparisons are made between the present results and corresponding results without transverse shear deformation taken into account. *E. Reissner* (Cambridge, Mass.)

2118:

**Gallely, G. D. Optimum design of thin circular plates on an elastic foundation.** Proc. Inst. Mech. Engrs. **173** (1959), 687–698.

2119:

**Serebryanyĭ, R. V. The deflection of a thin semi-infinite plate resting on an elastic layer of finite thickness.** Dokl. Akad. Nauk SSSR **125** (1959), 752–755 (Russian); translated as Soviet Physics. Dokl. **4**, 460–463.

2120:

**Slobodyanskiĭ, M. G. Bending of a plate of variable thickness.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 5, 99–108. (Russian)

2121:

**Muštari, H. M. Theory of bending of plates of moderate thickness.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 2, 107–113. (Russian)

The author derives a more exact theory of bending of isotropic plates. He takes into account all values of order $h^2/a^2$ and neglects in comparison with unity all values of order $h^4/a^4$ ($2h$—the thickness of the plate, $a$—its width). The author consistently introduces necessary corrections to the deformations and the stresses, and drops the classical assumption of the linear variation of the normal stresses along the thickness and parabolic variation of the shearing stresses. A numeral example (for $a=6h$) is given. The comparison of the results obtained using the author's theory and other approximate theories with the exact result indicates the superiority of the reviewed method. *Z. Kączkowski* (Warsaw)

2122:

**Lehnickiĭ, S. G. Theory of anisotropic thick plates.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 2, 142–145. (Russian)

The known theory of thick isotropic plates is generalized to transversally isotropic plates. The obtained equations and the corresponding formulas for the isotropic plates differ only in coefficients. The use of the classical theory of thin plates leads to greater errors in the case of transversal isotropy than in the case of isotropic material of the plate. *Z. Kączkowski* (Warsaw)

2123:

**Thorkildsen, R. L.; Hoppmann, W. H., II. Flexure of nonhomogeneous cylindrically aeolotropic plate.** J. Appl. Mech. **26** (1959), 669–672.

On a circular thin plate of cylindrical orthotropy and of elastic properties varying along the radius there acts a transverse load of central symmetry. Assuming that the coefficients in the differential equation can be expressed as convergent infinite power series, the author obtains the solution also in the form of infinite power series. Several particular cases are considered. *Z. Kączkowski* (Warsaw)

2124:

**Hieke, Max. Ein Beitrag zur Dynamik der Kreismembran.** Z. Angew. Math. Mech. **39** (1959), 476–483. (English, French and Russian summaries)

The differential equation for the displacement of a circular elastic membrane, clamped at the edge, is integrated for a suddenly applied load, which is uniformly distributed over a segment of the membrane and vanishes elsewhere. By appropriate superposition and limit processes, the author obtains solutions for a line load along a secant, a line moment along a secant, and a line load along a half-secant. The solution is carried out by the standard Fourier integral method. *H. J. Weinitschke* (Los Angeles, Calif.)

2125:

**Kil'čevskiĭ, N. A. Integrodifferential and integral equations of equilibrium of thin elastic shells.** J. Appl. Math. Mech. **23** (1959), 165–178 (124–133 Prikl. Mat. Meh.).

The paper deals first with the state of equilibrium of a circular arch under the action of a concentrated unit force, directed along a normal to the arch. This example clarifies certain particular points of the method, which then is extended to the treatment of the two-dimensional problem of the equilibrium of elastic shells of any form of the middle surface.

A special choice of a system of auxiliary displacements permits one to represent the displacement of a point of the middle surface of the shell by the sum of two displacements, namely the displacement of the corresponding point of the middle plane of a plate—the map of the middle surface of the shell—and a supplementary displacement depending in particular on the curvature of the middle surface of the shell. The application of the work reciprocity theorem for deriving integrodifferential and integral equations leads to the equilibrium of the shell in terms of displacements. As a simple example the equilibrium equations of a spherical dome are studied.

*W. Zerna* (Hannover)

2126:

**Berezovskiĭ, A. A. Integro-differential equations of the theory of thin shallow shells.** Ukrain. Mat. Ž. **11** (1959), 146–154. (Russian. English summary)

The systems of the differential equations of bending, vibrations and buckling of thin shallow shells are replaced by the author by systems of integro-differential equations. In certain particular cases the system reduces to a system of integral equations or to one integral equation of Fredholm's type of the second kind. For a simply supported rectangular shallow shell the author solves the integral equations making use of double trigonometric series. The results are in agreement with the known solutions of the corresponding differential equations. The practical application of the obtained system of integro-differential equations is limited to such cases in which the necessary Green functions are known.

*Z. Kączkowski* (Warsaw)

2127:

**Issler, Werner. Eine Kuppel gleicher Festigkeit.** Z. Angew. Math. Phys. **10** (1959), 576–578. (English summary)

Author's summary: "Thin shells of optimum strength are usually defined by the condition that two principal stresses in any point are equal to the yield stress. This definition is too restrictive; it is sufficient that principal stresses satisfy the yield condition of v. Mises or the one of Tresca.

"In this paper a proof is supplied for the existence of a spherical cupola of equal strength on the basis of v. Mises' yield condition, and an exact solution is presented for the distribution of thickness."

*W. Schumann* (Zürich)

2128:

**Malkina, R. L. Calculation of a non-circular closed cylindrical shell with arbitrary boundary conditions.** Inžen. Sb. **26** (1958), 25–38. (Russian)

2129:

**Oravas, Gunhard-Aestius. On the theory of nearly spherical thin shells.** Z. Angew. Math. Mech. **38** (1958), 379–386. (German, French and Russian summaries)

[This paper was listed in MR **20** #7424.]

It is proposed to solve the linear differential equations for rotationally symmetric deformations of thin shells of resolution which are "nearly spherical" in the sense that the meridian equation is of the form $r=\rho\sin\phi$, $z=\rho(1-\cos\phi)$ where $\rho=R[1+\mu f(\phi)]$ and $R$ is constant, by expansion in powers of $\mu$. The leading terms of this expansion are given in the paper.

*E. Reissner* (Cambridge, Mass.)

2130:

**Curkov, I. S. On elastic equilibrium of a rectangular panel of a shallow shell with finite deflections.** Inžen. Sb. **26** (1958), 49–53. (Russian)

2131:

**Fradlin, B. N.; Šahnovs'kiĭ, S. M. On the calculation of a rectangular depressed shell.** Dopovidi Akad. Nauk Ukraïn. RSR **1959**, 1202–1205. (Ukrainian. Russian and English summaries)

Author's summary: "A solution is obtained for a system of integral equations and the Green tensor components are thus determined for the problem of the equilibrium of a depressed rectangular shell."

2132:

**Klingbeil, E. Zur Theorie der Rotationsschalen vom Standpunkt numerischer Rechnungen.** Ing.-Arch. **27** (1959), 242–249.

This paper is concerned with some aspects of effective use of automatic computing machinery in the numerical solution of the equations of axisymmetrical (linear) bending of thin shells of revolution. It is noted that introduction of the arclength along a meridian as independent variable is particularly suitable insofar as both shallow and "steep" shells may be treated by one and the same scheme of numerical calculations. Three different sets of shell equations are then formulated with the arclength as independent variable: the Reissner-Meissner equations, their modified form due to Tölke and a set of shell equations recently given by Münz [Ing.-Arch. **19** (1951), 103–117, 255–270; MR **13**, 795]. The paper concludes with some valuable remarks on numerical treatment of these equations, but gives no examples. (Reviewer's note: It may be remarked that the arclength, in fact a more general meridian parameter, has also been used by E. Reissner [Reissner Anniversary Volume, Contributions to applied mechanics, pp. 231–247; Edwards, Ann Arbor, Mich., 1948; MR **11**, 69], leading to a form of shell equations suitable for calculations of both shallow and "steep" shells.) *H. J. Weinitschke* (Los Angeles, Calif.)

2133:

**Mazurkiewicz, Zbigniew. The boundary conditions and the equation of equilibrium and vibration for an anisotropic non-homogeneous plate.** Arch. Mech. Stos. **11** (1959), 729–735. (Polish and Russian summaries)

2134:

**Nevs'kiĭ, P. M. Free longitudinal vibrations of a string with elastic fastening of one end.** Dopovidi Akad. Nauk Ukraïn. RSR **1959**, 963–966. (Ukrainian. Russian and English summaries)

Author's summary: "The author considers the free longitudinal vibrations of a string, one end of which is fastened immovably and the other is fastened to a point

mass by elastic coupling. The vibrations of the string are investigated with allowance made for the velocity of propagation. Equations are derived for the displacement and changes in the tension of the string during vibrations."

2135:

**Sevin, Eugene. On the elastic bending of columns due to dynamic axial forces including effects of axial inertia.** J. Appl. Mech. **7** (1960), 125–131.

The elastic bending motion of an initially curved column compressed by a time-dependent axial force is discussed. The basic equations neglect the influence of the shear-strains and rotatory inertia forces, but take into consideration the axial inertia forces, though the numerical calculations show that their effects are usually negligible. *M. Sokołowski* (Warsaw)

2136:

**Böhme, H.; Fromm, E.; Sittig, E. Schwingungen des isotropen Kreiszylinders mit verschwindender Axialkomponente.** Acustica **10** (1960), 67–71. (English and French summaries)

Authors' summary: "Resonances of a type of elastic vibration in isotropic circular cylinders having vanishing axial component of motion are described. They are characterised by an index consisting of two integral numbers $(n, p)$. The series $(n, 1)$ degenerates with increasing $n$ into surface waves with wave fronts parallel to the axis. Measurements on glass and aluminium cylinders have confirmed the theoretical results which may also be used to interpret in principle Bergmann's photoelastic pictures."

2137:

**Piszczek, Kazimierz. A problem of dynamic stability of a prismatic bar with a mass attached to its end.** Rozprawy Inż. **7** (1959), 145–166. (Polish. Russian and English summaries)

The author considers a vertical bar with a large mass attached to one of its ends. Both ends of the bar, the one on a stationary support and the other at the moving mass, are either hinged or fixed. The forcing function is directed along the axis of the bar. In fact the arrangement described above serves as a model for a spring with a small pitch angle (5°–8°).

The author derives partial differential equations of motion from the Hamiltonian, the independent variables being the space variable $x$ and the time $t$. The Kirchoff assumption, that the deformation of the bar axis depends only on the time, leads to an expression for the potential energy which allows a formal solution in the case of fixed ends. Although the system is non-linear the author succeeds in separating the variables and replacing the original system by a system of ordinary differential equations. He neglects the damping but not the shearing forces. Only the fundamental mode is taken into account. The author obtains the critical force, finds three resonance zones and an additional resonance due to shearing forces. For steady vibrations he obtains the amplitude of the transverse motion. *T. Leser* (Aberdeen, Md.)

2138:

**Aronson, A. Ya. Influence of the surrounding mass of water on the oscillations of a rod immersed in it.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 5, 41–47. (Russian)

2139:

**Vasilenko, M. V.; Pisarenko, G. S. Forced bending-torsional vibrations in rods under the effect of internal energy dissipation.** Dopovidi Akad. Nauk Ukraïn. RSR **1959**, 833–836. (Ukrainian. Russian and English summaries)

Authors' summary: "The paper deals with the problem of forced bending-torsional vibrations of rods under the effect of internal energy dissipation.

"The asymptotical method of N. M. Kryloff and N. N. Bogoliuboff is applied for the solution of a system of differential equations of the problem."

2140:

**Mitra, A. K. A note on the torsional vibration of a conical bar with varying rigidity and density.** J. Tech. Bengal Engrg. Coll. **4** (1959), 37–39.

2141:

**Moseénkov, B. I. On the phenomenon of resonance under the effect of forces of its own weight of a double rigidity rotating rod.** Dopovidi Akad. Nauk Ukraïn. RSR **1959**, 959–962. (Ukrainian. Russian and English summaries)

Author's summary: "Transverse oscillations of a double rigidity rod in a transitional mode of rotation under the effect of forces of its own weight are considered when passing through the first critical number of angular velocity. A first approximation asymptotic solution is constructed, first approximation systems of equations are derived and analysed for this case, taking into consideration the forces of external resistance."

2142:

**Newman, M. K. Viscous damping in flexural vibrations of bars.** J. Appl. Mech. **26** (1959), 367–376.

2143:

**Bogdanoff, J. L.; Goldberg, J. E. On the Bernoulli-Euler beam theory with random excitation.** J. Aero/Space Sci. **27** (1960), 371–376.

From the authors' summary: "Mean square displacement and stress are calculated in a simply supported Bernoulli-Euler beam with distributed external viscous damping, for several types of random excitations. The mean square values are found to be finite except for two types of excitations which do not appear to have appreciable physical significance in engineering problems. The mean square calculus of random processes is used in the analysis." *R. A. Silverman* (New York)

2144:

**Hearmon, R. F. S. The frequency of flexural vibration of rectangular orthotropic plates with clamped or supported edges.** J. Appl. Mech. **26** (1959), 537–540.

2145:

**Cox, Hugh L.; Boxer, Jack. Vibration of rectangular plates point-supported at the corners.** Aero. Quart. **11** (1960), 41–50.

Finite difference expressions are used in conjunction with extrapolation procedures to obtain approximate values for the fundamental natural frequency of flexural vibration of uniform rectangular plates, having free edges and pinpoint supports at the four corners. For square plates, the lowest five frequencies and mode shapes are determined. *G. B. Warburton* (Edinburgh)

2146:

**Saharov, I. E. Dynamic rigidity in the theory of axisymmetric oscillations of circular and annular plates.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 5, 90–98. (Russian)

2147:

**Greenspon, Joshua E. Vibrations of a thick-walled cylindrical shell—comparison of the exact theory with approximate theories.** J. Acoust. Soc. Amer. **32** (1960), 571–578.

Numerical comparisons for frequencies and displacement component ratios for results of three-dimensional theory and corresponding results of various types of two-dimensional theories—membrane theory, classical bending theory, bending theory with transverse shear deformation taken into account—indicating the range of parameter values for which the various two-dimensional theories may be used safely. *E. Reissner* (Cambridge, Mass.)

2148:

**Hanin, Meir. High-frequency oscillation of slender bodies and wings.** J. Aero/Space Sci. **27** (1960), 544–545.

2149:

**Moiseev, N. N. On the theory of elastic oscillations of a fluid-filled body.** Dokl. Akad. Nauk SSSR **127** (1959), 51–54 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 806–809.

An elastic beam with straight-line axis of rigidity (and such that plane cross-sections remain plane) is partially filled with an inviscid incompressible fluid. The equations for infinitesimal torsional and flexural vibrations of axial planes are derived from Hamilton's principle, and under suitable assumptions it is shown that there is a complete system of eigen functions and that the eigen frequencies are unbounded, provided that the total potential energy is a positive definite functional. A Ritz method of solution is briefly formulated. *H. C. Levey* (Nedlands, Australia)

2150:

**Zanaboni, Osvaldo. Qualche considerazione sui fenomeni detti di instabilità progressiva.** Rend. Sem. Mat. Fis. Milano **28** (1959), 36–60. (English summary)

From the author's summary: "Progressive instability phenomena are synthetized and described, and some of their peculiarities are focused. The conclusion is reached that they are usually lability and not instability phenomena. Certain cases are also considered in which instability is only apparent and due to the hypothesis imposed; the configuration is, in effect, stable."

*R. M. Evan-Iwanowski* (Syracuse, N.Y.)

2151:

**Przemieniecki, J. S. Struts with linearly varying axial loading.** Aero. Quart. **11** (1960), 71–98.

Solutions are given for uniform, elastic struts under combined end compression and axial load uniformly distributed along the length. The case of distributed load applied on the deflected strut axis is considered, but special attention is given to the hitherto neglected and important case of load applied along the straight axis of the undeformed strut. Critical loads are given by curves for both cases for all combinations of fixed and supported ends, and formulas for deflections and bending moments are also given for numerous cases of axial loading of the second type combined with lateral loading or end moments. *L. H. Donnell* (Ann Arbor, Mich.)

2152:

**Vyaz'menskiĭ, S. P. On stability of the straight-line position of equilibrium of a compressed and twisted elastic rod.** Inžen. Sb. **25** (1959), 164–173. (Russian)

2153:

**Sanders, J. L., Jr. On the Griffith-Irwin fracture theory.** J. Appl. Mech. **27** (1960), 352–353.

The author brings out the implication of the Griffith fracture criterion, that fracture depends on conditions in the vicinity of the incipient crack.

*J. W. Craggs* (Newcastle-upon-Tyne)

2154:

**Swainger, Keith. ★Analysis of deformation. Vol. 4: Waves and vibrations.** The Macmillan Company, New York, 1959. xxvii + 370 pp. $15.00.

The chapter headings are: (1) Introductory; (2) Classical mathematical theories to analyse continua; (3) Waves through elastic solids of restricted isotropy; (4) Waves through fluids of restricted isotropy; (5) Waves through plastic solids of restricted isotropy; (6) Surface waves on elastic solids of restricted isotropy; (7) Surface waves on fluids of restricted isotropy; (8) Waves through visco-elastic solids of restricted isotropy; (9) Surface waves on visco-elastic solids of restricted isotropy; (10) Geophysics. There are also 7 appendices. Two alternative treatments are given, based respectively on the classical equations of motion and on the author's individualistic modification of these. Two more volumes are promised.

*F. Ursell* (Cambridge, England)

2155:

**Miles, John W. Homogeneous solutions in elastic wave propagation.** Quart. Appl. Math. **18** (1960/61), 37–59.

Techniques traditionally associated with the method of conical flows are applied to two-dimensional problems involving the diffraction of elastic waves by a semi-infinite half-plane (representing either a free or a rigid surface) within a uniform isotropic elastic solid. The presence of this surface ensures coupling between shear and

dilation waves, and it is in this fairly complicated physical situation that the author makes his analysis. He takes an incident plane pulse and calculates the corresponding scattered disturbance without the use of integral transforms.

The method used has the advantage of separating out the waves which are reflected and refracted at the surface from the disturbance set up by diffraction at the edge of the half-plane, and in finding this diffracted field an interesting use is made of complex function theory.

Similar techniques have been used independently both by J. D. Craggs and by the reviewer.

*V. M. Papadopoulos* (Melbourne)

2156:

**Jones, R. P. N. The generation of torsional stress waves in a circular cylinder.** Quart. J. Mech. Appl. Math. **12** (1959), 325–336.

This paper is concerned with the application of the exact elastic equations to the problem of the generation of torsional waves in a cylindrical bar when the bar is subjected to prescribed boundary conditions. A general solution in series form is obtained which is in effect an extension of the Pochhammer-Chree treatment for harmonic waves. The particular problem of a suddenly applied torque in the form of a distributed shear acting circumferentially on the surface of the bar is then treated. It is shown that at large distances along the bar the surface displacements (except close to the wave front) are described satisfactorily by the elementary theory which takes into account only the fundamental mode of propagation. The expressions for the velocities and stresses, however, do not converge satisfactorily where a rapidly applied stress is postulated and an alternative method of treatment in terms of wave reflections at the free surface of the bar is discussed. *H. Kolsky* (London)

2157:

**Duff, G. F. D. The Cauchy problem for elastic waves in an anisotropic medium.** Philos. Trans. Roy. Soc. London Ser. A **252** (1960), 249–273.

Using Fourier transforms the author considers elastic waves produced in an anisotropic medium by a local initial disturbance. The solution consists of a continuous wave which lasts for a definite period of time, and a number of sharp waves. The continuous wave may arrive at a field point in advance of the first sharp wave, though the disturbance will always terminate with the last sharp wave. The solution appears as the sum of three modes which correspond to the three sheets of a certain wave surface. The paper closes with illustrations for particular anisotropic media. The calculations presented should serve as a foundation for the study of time-dependent elastic waves. *A. E. Green* (Newcastle-upon-Tyne)

2158:

**Brehovskih, L. M. Propagation of surface Rayleigh waves along the uneven boundary of an elastic body.** Akust. Ž. **5** (1959), 282–289 (Russian); translated as Soviet Physics. Acoust. **5** (1960), 288–295.

Author's summary: "The damping of a surface Rayleigh wave due to its scattering by roughness of the surface is considered. The problem is solved in the first approximation. The height of the surface roughness is assumed small in comparison with the wavelength. The results of the calculations show that with definite values of the spatial period of the unevenness the damping due to scattering, even by fine-grained nonuniformities, can be very great."

2159:

**Babič, V. M. Propagation of non-stationary waves and caustics.** Leningrad. Gos. Univ. Uč. Zap. Ser. Mat. Nauk **32** (1958), 228–260. (Russian)

2160:

**Mitra, Manindra. Exact transient solution of the buried line source problem for an asymmetric source.** Z. Angew. Math. Phys. **9a** (1958), 322–331. (German summary)

Following Garvin [Proc. Roy. Soc. London Ser. A **234** (1956), 528–541; MR **17**, 1158], Cagniard's method is used to determine the surface displacement components in the case of an impulsive dilatational source buried in a semi-infinite medium. The radial displacement consists of two sudden jerks in opposite directions, each followed by a gradual incomplete recovery. *S. C. Das* (Madras)

2161:

**Kaliski, S.; Kurlandzki, J. Cauchy's problem for a transversally isotropic elastic body.** Arch. Mech. Stos. **10** (1958), 825–838. (Polish and Russian summaries)

The problem of radiating waves in a transversely isotropic, elastic body was considered by Carrier [Quart. Appl. Math. **4** (1946), 160–165; MR **8**, 120] subject to the restriction that only four of the five constants of elasticity are independent. The restriction is removed in the present paper. *R. D. Mindlin* (New York)

2162:

**Kaliski, Sylwester. On a conception of basic solutions for orthotropic elastic and anelastic bodies.** Arch. Mech. Stos. **11** (1959), 45–60. (Polish and Russian summaries)

The author employs the result that the modes of free vibration of an orthotropic, elastic, rectangular parallelepiped are simple products of trigonometric functions in rectangular coordinates if the normal displacement and tangential traction vanish on the faces. Since the product functions form an orthogonal set, Fourier series solutions may be constructed readily for initial value problems of radiating waves. He points out that these solutions are applicable to bodies of any shape and boundary conditions during the time before the first wave strikes the boundary. Extensions to curvilinear orthotropy and viscoelasticity are indicated.

*R. D. Mindlin* (New York)

2163:

**Kaliski, S.; Kurlandzki, J. The basic solution and Cauchy's problem for a paratropic body.** Arch. Mech. Stos. **11** (1959), 61–70. (Polish and Russian summaries)

The method of the preceding paper is applied to

Kaczkowski's "paratropic" elastic medium [Arch. Mech. Stos. 7 (1955), 52–86; MR 17, 210]. In this case the coordinate system and the parallelepiped are oblique.

*R. D. Mindlin* (New York)

2164:

**Guseĭn-Zade, M. I.; Kuzin, P. A. Action of an impulsive load on an elastic layer lying on a liquid elastic half-space.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 64–72. (Russian)

To integrate the system of three wave equations for wave-potentials describing the motion of the elastic layer and the liquid medium, the double Mellin-Laplace transforms are used. The rigorous solution, involving contour integrals applied to infinite series, has been transformed to a form suitable for numerical calculations. Using these final formulae the values of displacements and stresses at some characteristic points shortly after the impulsive load has been applied can be comparatively easily evaluated.

*M. Sokołowski* (Warsaw)

2165:

**Naghdi, P. M.; Orthwein, W. C. Response of shallow viscoelastic spherical shells to time-dependent axisymmetric loads.** Quart. Appl. Math. **18** (1960/61), 107–121.

The paper deals with time-dependent primarily transverse deformations of unlimited shallow, homogeneous, isotropic, spherical shells. As for the corresponding problems of the elastic shell it is possible, through justified neglect of longitudinal inertia, to reduce the problem to two simultaneous equations for Airy's stress function $F$ and transverse deflection $w$. The solutions are obtained through joint use of Laplace and Hankel transforms and, by interchanging the order of the inversions, there is avoided the otherwise intricate task of contour integration in the complex Laplace transform-plane. Explicit results in integral form are deduced for shells under instantaneous pulse loading and are particularized to the cases of Maxwell and Kelvin solids.

*E. Reissner* (Cambridge, Mass.)

2166:

**Eason, George; Shield, Richard T. The plastic indentation of a semi-infinite solid by a perfectly rough circular punch.** Z. Angew. Math. Phys. **11** (1960), 33–43. (German summary)

The incipient indentation of a non-hardening rigid-plastic solid by a flat punch of circular section is treated. The Tresca yield function and plastic potential are assumed, and a particular complete solution is obtained. This corresponds to a special variation of the coefficient of friction (between 0 and 0.139) along the punch surface, such that the material adjoining and near the punch is all stressed plastically. (The title description 'perfectly rough' is misleading.)

For this stress field a particular virtual velocity mode is proposed (infinitely many are possible). In this a cap of rigid material is pushed in front of the punch. This mode satisfies the Haar-Karman hypothesis. The question, for what increments in the boundary conditions this mode is actual, is not examined.

It should also be pointed out that the region below the punch is not necessarily rigid, and the authors' deduction (§ 5, last sentence) from the theorem of Bishop, Green and Hill is invalid. Again, for other frictional conditions (in particular when the friction is equal to the shear yield stress) different stress solutions would be obtained, to each of which would correspond a family of virtual modes (some involving rigid caps of various shapes). The paper does not make clear the very special character of the proposed solution.

*R. Hill* (Nottingham)

2167:

**Ivlev, D. D. A particular solution of the general equations of the theory of ideal plasticity in cylindrical coordinates under the Tresca yield condition.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 132–133. (Russian)

Stress equations are given, in cylindrical coordinates $\rho$, $\theta$, $z$, in general form for Tresca material, assuming that the stress field does not depend on $\theta$. The Haar-von Karman assumption concerning the values of two principal stresses makes the problem a statically determinate one. Differential equations of the corresponding velocity field are sketched. No solution of any boundary value problem is given. Spiral plastic flow under torsional indentation of a rough circular die is mentioned as an example.

*A. Sawczuk* (Warsaw)

2168:

**Hodge, P. G., Jr. Yield conditions for rotationally symmetric shells under axisymmetric loading.** J. Appl. Mech. **27** (1960), 323–331.

Shells of nonhardening rigid/plastic material are considered, with identical yield function and plastic potential. For the axially symmetric problem the yield function is representable as a surface in the four-dimensional space of the stress resultants. Approximating surfaces are compared with the exact surface for the Tresca criterion. A new approximation is proposed on the basis that in most shell problems the moments and direct forces are not of simultaneous importance, and so their interaction can reasonably be neglected. A method helping the visualization of the 3-dimensional surface is described. Finally, improved upper and lower bounds are obtained for the collapse load of a spherical cap, either clamped or simply supported.

*R. Hill* (Nottingham)

2169:

**Nagai, Tamotsu. Large plastic deformations of corrugated beams under transverse impact.** J. Zosen Kiokai **106** (1960), 181–187. (Japanese. English summary)

2170:

**Ulrich, E. Über das Problem der Vergleichsspannungen in der Festigkeitslehre.** Forsch. Gebiete Ingenieurwesens **25** (1959), 106–114.

2171:

**Feinberg, S. M. The principle of limit stress.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 4, 101–111. (Russian)

2172:

**Castiglioni, Alfredo. Travi elasto-plastiche a plasticità diffusa.** Ist. Lombardo Accad. Sci. Lett. Rend. A **93** (1959), 548–564.

The author considers the motion of yield hinges during the steady increase of load on the beam. He seems unaware of the work of P. S. Symonds and his collaborators.

*D. R. Bland* (Manchester)

2173:

**Kalnins, A. On plane-stress solution of a compressible wedge with the use of Mises' yield condition.** J. Appl. Mech. **26** (1959), 676–678.

Stresses and displacements are found throughout the wedge. Unlike the Tresca case, the plastic zones on the outside of the wedge do not begin to form at the same applied pressure and, at higher pressures, are not symmetrical about the internal bisector of the wedge.

*D. R. Bland* (Manchester)

2174:

**Storchi, Edoardo. I problemi ristretti in plasticità.** Rend. Sem. Mat. Fis. Milano **29** (1959), 103–152. (English summary)

The paper is a review of the author's research in plane plasticity; see Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **22** (1957), 286–293; (8) **23** (1957), 45–51; (8) **24** (1958), 685–692; Ist. Lombardo Sci. Lett. Rend. Cl. Sci. Mat. Nat. **91** (1957), 778–788; Ist. Lombardo Accad. Sci. Lett. Rend. A **92** (1957/58), 173–205; 321–348 [MR **19**, 1113; **20** #3686, 7447, 2131; **21** #2423a, b].

*D. R. Bland* (Manchester)

2175:

**Krasovskiĭ, Yu. P. The solvability of the plane problem in the theory of small elastic-plastic deformation.** Dokl. Akad. Nauk SSSR **126** (1959), 961–963 (Russian); translated as Soviet Physics. Dokl. **4**, 705–707.

2176:

**Vorovič, I. I.; Krasovskiĭ, Yu. P. On the method of elastic solutions.** Dokl. Akad. Nauk SSSR **126** (1959), 740–743 (Russian); translated as Soviet Physics. Dokl. **4**, 701–704.

2177:

**Kusuda, Tadao. On the plastic buckling of stiffened panels under axial compression. I. Panels with axisymmetric type of stiffeners.** J. Zosen Kiokai **105** (1959), 137–146. (Japanese. English summary)

Author's summary: "The general instability of a stiffened flat plate under axial compression is a problem of primary buckling of a bending type of deformation of the stiffener about an axis parallel to the plane of the sheet and a twisting type of deformation in which the stiffener rotates about an axis in the plane of the sheet. The paper treats the primary buckling of stiffened plates in the strain-hardening range as well as in the elastic range. The secondary or local buckling of the stiffener or plate such as crippling of the stiffener or interfastener buckling of plate are excluded.

"From consideration of the yielding process of structural steel it has been shown that the material becomes anisotropic when strained into the strain-hardening range. Introducing appropriate moduli the problem may then be treated as buckling of an orthotropic plate.

"Buckling in the elastic range, then, is a particular case of the orthotropic solution with the elastic moduli of steel. The application of the integral equation to the buckling of a stiffened plate simplifies the problem and saves labor and time for computation. The single basic integral equation covers the whole domain of the plate. The stiffener reactions enter as transverse or twisting loads but do not introduce new boundary conditions.

"The required minimum bending rigidity of a stiffener is obtained in both elastic and strain-hardening range. The effect of torsional resistance of a stiffener on buckling strength is investigated for the stiffener with a symmetric type of thin-walled open cross-section."

2178:

**Kusuda, Tadao. On the plastic buckling of stiffened panels under axial compression. II. Panels with unsymmetric type of stiffener.** J. Zosen Kiokai **106** (1960), 171–179. (Japanese. English summary)

Author's summary: "In the previous paper [#2177] the buckling of a stiffened panel with a symmetric type of stiffener was treated as an eigenvalue problem of an orthotropic plate with proper moduli of materials in the strain-hardening range. The equation of equilibrium of a stiffener can be generalized for an arbitrary shape of thin-walled open cross-section attached to a plate if proper consideration is given to the mutual interaction between plate and stiffener. If a beam with an arbitrary cross-section is subjected to transverse load, the beam is bent and twisted simultaneously unless the line of loading passes through the shear center. When a beam is attached to a plate, the enforced axis causes twisting of the stiffener if it is bent, or the other way around, bending of the stiffener if it is twisted. Therefore the bending and the twisting of the stiffener are no longer separable for such a stiffener with an unsymmetric cross-section.

"When the plate starts to buckle, the distributed resistance due to the longitudinal and/or transverse stiffener can be obtained from the equation of equilibrium of a stiffener attached to the plate. The integral equation for the buckling of a longitudinally and transversely stiffened plate can be formulated as in the previous paper. The buckling strength of a stiffened plate is obtained as an eigenvalue of the integral equation. From the computation given in this paper it may be concluded that a longitudinal stiffener is tolerably effective to prevent the buckling of a plate compared to a transverse stiffener, and the efficiency of an inverted angle stiffener is considerably lower than that of a Tee stiffener when the stiffener is subjected to axial loads as in a longitudinal stiffener. In other words, the existence of axial load weakens the torsional resistance of a stiffener with an unsymmetric open cross-section as well as the bending resistance of the stiffener, and its influence for an unsymmetric cross-section is considerably greater than for a symmetric one. The results of this paper, together with the previous one, are used to specify the proper geometric conditions of the stiffened plate such that each panel may develop large plastic deformation without buckling and a consequent fall-off in load. These requirements are essential for a successful application of plastic design methods to plate structures."

2179:

**Bleich, H. H.; Dillon, O. W., Jr. Nonlinear creep deformations of columns of rectangular cross section.** J. Appl. Mech. **26** (1959), 517–525.

2180:

**Rozovskiĭ, M. I. Non-linear integral-operator equations of creep and the torsion problem for a cylinder with large angles of twist.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 5, 109–116. (Russian)

2181:

**Nowinski, Jerzy. Thermoelastic problem for an isotropic sphere with temperature dependent properties.** Z. Angew. Math. Phys. **10** (1959), 565–575. (German summary)

It is well known that, in an incompressible sphere subjected to spherically symmetric temperature distribution, radial displacement and hence strain components are independent of the stress-strain law and can be represented in a simple form. Stresses are then found by introducing the particular stress-strain law.

The author assumes Hooke's law with temperature-dependent coefficients and works out a numerical example.

*H. Parkus* (Vienna)

2182:

**Sneddon, I. N.; Lockett, F. J. On the steady-state thermoelastic problem for the half-space and the thick plate.** Quart. Appl. Math. **18** (1960/61), 145–153.

This paper concerns the determination (within classical elasticity theory) of the steady-state thermal displacements and stresses induced by a given surface temperature in a semi-infinite body bounded by a plane or in a body bounded by two parallel planes. Solutions to the foregoing problems were obtained previously, with the aid of the method of Green's functions, by Sternberg and McDowell [Quart. Appl. Math. **14** (1957), 381–398; MR **19**, 343] for the case of the half-space, and by McDowell [Proc. Third Midwest. Conf. Solid Mech., 1957, pp. 72–85, Univ. Mich. Press, Ann Arbor, Mich., 1957; MR **19**, 1116] for the case of the infinite slab. Both problems were also treated by Muki [Proc. Fac. Engrg. Keio Univ. **9** (1956), 42–62; MR **19**, 792] by means of a combined Fourier-series and Hankel-transform technique. In the present paper the foregoing problems are reconsidered on the basis of a double Fourier-transform scheme. The authors' claim that their method of solution is much more suitable for the discussion of special problems, appears to be in need of qualification.

*E. Sternberg* (Providence, R.I.)

2183:

**Landau, H. G.; Weiner, J. H.; Zwicky, E. E., Jr. Thermal stress in a viscoelastic-plastic plate with temperature-dependent yield stress.** J. Appl. Mech. **27** (1960), 297–302.

The problem considered is that of a viscoelastic, perfectly plastic plate which is subjected to a temperature distribution depending on time and the thickness coordinate. Viscosity effects and the (von Mises) yield criterion are taken in temperature-dependent form, the other properties of the material being assumed independent of temperature. Equations for determination of the stress distribution are obtained in a form suitable for numerical computation, and are demonstrated by calculation of the residual stress in a symmetrically cooled steel plate. In this case it is found that viscoelasticity has little effect, but the temperature-dependence of the yield stress is important. Growth of the plastic regions follows a pattern similar to that in the elastic-plastic case with temperature-independent properties.

*F. J. Lockett* (Sevenoaks)

2184:

**Parkes, E. W. Effects of repeated thermal loading. The influence of the variation of strength with temperature on structural behaviour.** Aircraft Engrg. **32** (1960), 222–229.

Author's summary: "A very simple redundant structure is subjected to temperature cycling, primarily to determine the influence of the yield stress/temperature relation on its behaviour: the range and periodic time of the temperature cycle are included as subsidiary variables.

"It is found that improving the strength of the material at elevated temperatures may have the undesirable effect of hastening incremental collapse of the structure, and that the most rapid incremental collapse is not necessarily associated with maximum values for the range and periodic time of the temperature cycle.

"It is also found that the common assumption that the strength of the material is independent of temperature may in some circumstances be ambiguous, since there may be a sudden discontinuity in behaviour between a structure made from a material having a slight negative strength/temperature gradient and one made from a material having a slight positive gradient."

2185:

**Chen, L. H. Piping flexibility analysis by stiffness matrix.** J. Appl. Mech. **26** (1959), 608–612.

From the author's summary: "A method of analysis is formalized for the solution of thermal-expansion stress problems in piping systems."

*M. Sokołowski* (Warsaw)

2186:

**Takizawa, É. I. On the thermoelastic waves in liquids.** Acustica **10** (1960), 25–29. (French and German summaries)

Author's summary: "For a system showing relaxation phenomena there is calculated, for the isothermal case of small relaxation coefficients, the specific heat and thermal conductivity according to both the Debye and the Brillouin theories. The theoretical results agree, in the special case of a system having shear viscosity, closely with the expressions obtained by Lucas."

## STRUCTURE OF MATTER

2187:

**Wannier, Gregory H. ★Elements of solid state theory.** Cambridge University Press, New York, 1959. vii + 270 pp. $6.50.

The technological importance of solids and solid state

devices has led to the expenditure of a great amount of scientific energy in this field, and to the growth of a correspondingly vast and detailed literature. The theory of solids in its present form is a branch of applied quantum mechanics, and much of the theory has been concerned with the finer details of the physical behavior of solids rather than with fundamental principles. This makes it difficult for the beginner or the interested general reader to obtain a balanced view of the field.

The present book is a very attractive addition to the textbook literature on solid state theory. Its outstanding quality is the balanced judgment with which the author controls both the physical and mathematical sides of his subject. The material covered is sufficiently extensive to give a real insight into the nature of the solid state, and there is little here that the reader will need to unlearn or forget if he goes further into the subject. In addition to the older topics of crystal structure, lattice dynamics, and the theory of one-electron energy states in a periodic lattice, quite a number of newer topics such as cooperative phenomena, Ising lattices, and semiconductors, are included.

The principal qualifications required of the reader are covered by a variant of the formula familiar to mathematicians—a reasonable acquaintance with classical theoretical physics and elementary quantum mechanics, and a modicum of the indispensable quality of physical intuition. *E. L. Hill* (Minneapolis, Minn.)

2188:

**Zel'dovič, Ya. B. Energy levels in a distorted Coulomb field.** Fiz. Tverd. Tela **1** (1959), 1637–1641 (Russian); translated as Soviet Physics. Solid State **1** (1960), 1497–1501.

In the study of impurity semiconductors it is of importance to estimate the distortion of the wave function of an electron in the neighborhood of a donor ion. The author formulates the following idealized problem. Consider a spherically symmetric potential field which is of Coulomb type beyond the radius of the ion, but which can be of general shape in its interior. It is desired to study the energy levels and wave function of the Schrödinger equation for an electron in this potential field. This problem has been discussed many times by use of special models and of perturbation theory. The author studies it directly by formal integration procedures applied to the radial equation. *E. L. Hill* (Minneapolis, Minn.)

2189:

**Peretti, Jean. Some remarks about frequency spectra of crystal lattices.** Phys. and Chem. Solids **12** (1959/60), 216–232.

Author's summary: "The general construction of the frequency distribution of a crystal lattice when the force constants between atoms are known is treated. The intermediate step is to consider a function $F(z)$, of the complex variable $z$, which is expressible in terms of the force constants in a compact form. The frequency spectrum $g(x)$ itself is, apart from a multiplicative constant, the imaginary part of $F(z)$.

"This general formalism is applied, first, to calculate $g(x)$ for special lattices; secondly, to investigate the singularities of the spectrum; thirdly, to derive a low-temperature expansion of the spectrum; and fourthly, to investigate the analytical nature of one-dimensional frequency spectra."

2190:

**Seiden, Joseph. Ferromagnétisme d'un cristal imparfait à l'approximation d'Ising.** J. Phys. Radium **20** (1959), 876–889. (English summary)

Author's summary: "Theoretical study of the influence of lattice defects on thermodynamic properties of ferromagnetic materials. The theory of the Ising model for an imperfect square lattice is expounded. The partition function of the imperfect square lattice is calculated rigorously to the first order in $c$ ($c$ is the concentration of lattice defects). It is proved that the theory can also be applied to the imperfect cubic lattice. It is shown that when $c < 0.1$, the phase transition of the imperfect crystal is of the same type as the transition of the perfect crystal for a large class of lattice defects, and a simple formula for the Curie temperature of the imperfect lattice is obtained. The theory is compared with experiments on the Curie temperature of alloys of nickel with a non-ferromagnetic metal. The theory gives the correct order of magnitude, but the Ising model, because it neglects the band structure of electronic spectra in metals, cannot provide a wholly satisfactory quantitative treatment of ferromagnetic alloys in the vicinity of their Curie temperature."

2191:

**Bumble, S.; Honig, J. M. Utilization of order-disorder theory in physical adsorption. I. Fundamental equations.** J. Chem. Phys. **33** (1960), 424–431.

Authors' summary: "The order-disorder theory of Hijmans and de Boer is adapted to a description of equilibrium between a gas and a phase forming a localized submonolayer on an energetically uniform surface. Isotherm equations are derived for adsorption in the absence of lateral interactions, and in the presence of nearest- and next-nearest-neighbor interactions. The Langmuir and Fowler-Guggenheim equations result as special cases of the present treatment. The general theory is then applied to adsorption processes on sites in hexagonal configurations. A discussion of the resulting isotherm equations is presented."

2192:

**Gourary, B. S.; Maradudin, A. A. Adsorption and emission spectra of an electron in a one-dimensional deep trap.** Phys. and Chem. Solids **13** (1960), 88–104.

Authors' summary: "The wave functions and the energies of the bound states of an electron in a special one-dimensional deep trap are calculated. Only two essential approximations are made in the course of the treatment, namely the Born-Oppenheimer approximation for the separation of the electronic and the nuclear motions, and the harmonic approximation for the lattice energy. Further approximations are also made, but these are not essential to the calculation and they can be avoided, if necessary. Green's function techniques are employed in the solution of the electronic and the lattice-vibration

problems, and they make possible the treatment of both the localized and the extended lattice-vibration modes and their influence on the electronic absorption (emission) spectrum. The first two moments of the absorption and emission spectra are then calculated. The principal utility of this work lies in providing a test case for more approximate theories, although a generalization of the method to three dimensions and to more realistic potentials should also be possible."

2193:

**Hebborn, J. E.; Sondheimer, E. H. The diamagnetism of conduction electrons in metals.** Phys. and Chem. Solids **13** (1960), 105–123.

Authors' summary: "The partition function is calculated to order $H^2$ for a gas of independent electrons interacting with a periodic lattice potential and a uniform applied magnetic field $H$. A formula is deduced for the steady diamagnetic susceptibility of a metal, expressed in terms of the Bloch wave functions and energy levels. The result is exact and general within the limits of the Bloch model."

2194:

**Wahl, F. Klassisch atomistische Energieberechnung null- und eindimensionaler Gitterstörungen.** Z. Naturforschg. **15a** (1960), 616–625.

Author's summary: "Zur Berechnung der Verzerrungsenergie von Störstellen, die man mit Hilfe der klassischen nichtlinearen Gitterstatik behandelt, wird eine Methode angegeben. Sie besteht in der Ersetzung des quadratischen Verzerrungsanteils im Energieausdruck durch die höheren Kraftglieder, was sich zufolge des Kräftegleichgewichts zwischen linearer und nichtlinearer Gitterreaktionen durchführen läßt. Da die nichtlinearen Gitterreaktionsanteile räumlich auf wenige, ausgezeichnete Bereiche beschränkt sind, wird das Problem der Summation der Teilchenpotentiale über das gesamte Verschiebungsfeld reduziert auf eine Summation, welche nur die Freiheitsgrade jener Stellen enthält, an denen die Nichtlinearitäten für den Gleichgewichtszustand wesentlich werden. Bei nulldimensionalen Störstellen sind diese auf die Umgebung der Störstellen allein konzentriert. Die Betrachtungen verlaufen hier sehr einfach. Bei eindimensionalen Störstellen spielen die Vorverschiebungen eine wesentliche Rolle, so daß die Nichtlinearitäten zunächst nicht ohne weiteres am Versetzungskern allein angenommen werden dürfen. In diesem Fall lassen sich die Betrachtungen nur am speziellen Modell anstellen. Als Modelle werden Schrauben- und Stufenversetzungen untersucht."

2195:

**Rice, Stuart A. Dynamical theory of diffusion in crystals. I.** Phys. Rev. (2) **112** (1958), 804–811.

Author's summary: "The theory of self-diffusion in a crystal via the vacancy mechanism is investigated from a detailed dynamical model. It is shown that the parameters which determine the diffusion coefficient can be defined in terms of the normal coordinates of the crystal. The effects of lattice imperfections are considered explicitly in the formulation in the normal mode analysis, but no detailed analysis is given. The effects of the correlation in motion of atoms is considered and some comments on the physical nature of the contributions to the enthalpy and entropy of activation are presented."

2196:

**Rice, Stuart A.; Nachtrieb, Norman H. On the dynamical theory of diffusion in crystals. II. Pressure dependence of the self-diffusion constant.** J. Chem. Phys. **31** (1959), 139–145.

Authors' summary: "The origin of irreversibility in the phenomenon of crystal diffusion is examined from a dynamical point of view. It is shown that straightforward considerations lead to the establishment of two time scales, one for jumps between lattice sites and one for vibrations about a given lattice point. The dissipation of energy is shown to occur almost entirely by relaxation of 'recaptured' particles, little energy being lost in the transit across the barrier. The quasi-equilibrium and non-equilibrium formulations are compared and shown to be quite similar in structure and functional dependence."

2197:

**Lawson, A. W.; Rice, Stuart A.; Corneliussen, Roger D.; Nachtrieb, Norman H. On the dynamical theory of diffusion in crystals. III. Some model calculations and relation to continuum theory.** J. Chem. Phys. **32** (1960), 447–455.

Authors' summary: "A dynamical theory of diffusion developed previously is extended to the study of the pressure dependence of the self-diffusion constant. It is shown that in the limit of low pressures there exists a law of corresponding states of the form $\ln \mathcal{D} \propto T_m/T$, where $T_m$ is the melting temperature of the crystal. It is further shown that the temperature and pressure derivatives of the self-diffusion coefficient are related as $\Delta H\dagger = (\Delta H_m/\Delta V_m)\Delta V\dagger$ with a dagger referring to activated quantities and a subscript $m$ to fusion parameters. Application of the theory is made to Na and Pb. For Pb, the computed heat of activation is 24,300 cal to be compared with the experimental value of 24,210 cal."

2198:

**Rice, Stuart A.; Frisch, Harry L. Dynamical theory of diffusion in crystals. IV. Some aspects of the introduction of irreversibility.** J. Chem. Phys. **32** (1960), 1026–1034.

Authors' summary: "Some simple models are considered which permit exact evaluation of the various dynamical parameters of the theory of crystal diffusion previously proposed. The models considered suggest that: (a) The jump frequency is insensitive to the normal mode modifications produced by the adjacent vacancy. (b) There is considerable relaxation about a vacancy. (c) The volume of formation of a vacancy is related to the molar volume by $\Delta V^* \approx V/(\gamma_G - \frac{1}{3})$, where $\gamma_G$ is the Gruneisen constant.

"The restrictions of the models are briefly analyzed, and the mass dependence proposed by Bak and Prigogine is compared with the quasi-equilibrium model and the discrepancy discussed."

2199:

**Butcher, P. N. An introduction to the theory of solid-state masers. With particular reference to the travelling-wave maser.** Proc. Inst. Elec. Engrs. B **107** (1960), 341–353.

Author's summary: "The paper is an introduction to the theory of solid-state masers with particular reference to the travelling-wave maser. The relevant properties of paramagnetic ions are described and the quantum theory of maser action is outlined qualitatively. A semi-classical treatment is developed which is based on the classical equation of motion of a magnetic dipole. It is used to evaluate the engineering characteristics of a travelling-wave maser which employs the comb type of slow-wave guide."

2200:

**Karle, J.; Hauptman, H. A unified program for phase determination, type $3P_2$.** Acta Cryst. **13** (1960), 469–476.

Authors' summary: "The unified program for phase determination, valid for all the space groups and both the equal and unequal atom cases, is continued here. The present paper is concerned with the centrosymmetric space groups comprising type $3P_2$. A detailed procedure for phase determination is described for this type."

## FLUID MECHANICS, ACOUSTICS

See also A1732, A1733, 1981, 2011, 2018, 2149, 2154, 2290.

2201:

**Lighthill, M. J. Mathematics and aeronautics.** J. Roy. Aero. Soc. **64** (1960), 375–394.

2202:

**Gallissot, F. Les formes extérieures et la mécanique des milieux continus.** Ann. Inst. Fourier. Grenoble **8** (1958), 291–335.

The author extends his work on the use of exterior forms in mechanics [same Ann. **4** (1952), 145–297; MR **15**, 836] to mechanics of the continuum. Noting that a motion of a continuum in 3-space $R^3$ amounts to a map of $R^4 = R^3 \times$ time axis into $R^3$, he generalizes to $C^r$-maps of a manifold $V_p$ into a manifold $W_n$. The space $J^1(V_p, W_n)$ of jets carries a natural $(p+1)$-form $\phi_{p+1}$ (once a volume element has been selected in $V_p$). The main objects of discussion are then the "generating forms" $\Omega_{p+1} = \phi_{p+1} + \omega \wedge V_p'$, where $V_p'$ is the form induced by the volume element of $V_p$ and where $\omega$ is any 1-form. The solutions of the associated exterior system (satisfying some additional conditions) are also solutions of a system of first order partial differential equations, generalizing the Hamilton-Jacobi equation. Often one has a group $G$ operating on $V_p \times W_n$, and the form $\Omega_{p+1}$ is more or less determined by the requirement of invariance under $G$. The particular case of the Galilean group is studied in detail. As examples there are treated the mechanics of threads, the Navier-Stokes equations in $n$ dimensions, the specialization to rigid systems. The author points out that in his approach one needs no notion of constraint, nor, for rigid systems, of "interior forces."

*H. Samelson* (Princeton, N.J.)

2203:

**Ivanilov, Yu. P.; Moiseev, N. N.; Ter-Krikorov, A. M. On the asymptotic character of Lavrentiev's formulas.** Dokl. Akad. Nauk SSSR **123** (1958), 231–234. (Russian)

Consider the steady two-dimensional potential flow of a homogeneous fluid in the infinite strip with boundary streamlines $y=0$ and $y=f(x)$ $(-\infty < x < \infty)$, where the stream function $\psi(x, y)$ satisfies the boundary conditions $\psi(x, 0) \equiv 0$, $\psi(x, f(x)) \equiv 1$. The author uses the method of expansion in terms of a small parameter $\varepsilon$, as is done for example in the Rayleigh-Janzen method, to obtain formal expansions

$$\psi(x, y) = \sum_{i=1}^{\infty} \psi_i(x, y), \quad \varphi(x, y) = \sum_{i=1}^{\infty} \varphi_i(x, y),$$

where $\varphi(x, y)$ is the velocity potential. He then states without proof that a necessary and sufficient condition for $\sum_{i=1}^{m} \psi_i(x, y)$ to be an asymptotic representation of $\psi(x, y)$ is that $f^{(k)}(x) = O(\varepsilon^{l_k})$ $(k = 1, 2, \cdots, 2m)$, wherein $l_k$ are arbitrary positive numbers. The fluid velocity on the boundary $y=f(x)$ is then shown to be approximately $(1 + \frac{1}{2}ff'' - \frac{1}{4}f'^2)/f$, which reduces to the Lavrent'ev formula $(1 + \frac{1}{2}ff'')/f$ when $f = O(\varepsilon)$, $f' = O(\varepsilon^{3/2})$, $f'' = O(\varepsilon)$ and terms beyond $O(\varepsilon^2)$ are neglected. [See M. A. Lavrent'ev, Akad. Nauk Ukrain. RSR. Zbirnik Prac' Inst. Mat. **1946**, no. 8, 13–69 = Amer. Math. Soc. Transl. No. 102 (1954); MR **14**, 102; **15**, 906.]

The author also generalizes the flow problem by replacing the boundary $y=0$ by the arbitrary boundary $y=g(x)$ and treats in addition an axially symmetric flow. He concludes by applying his results to two specific flow problems previously treated elsewhere. The reader intent on using the results should be warned that the equations immediately following equation (8) contain numerous errors.

*J. F. Heyda* (Cincinnati, Ohio)

2204:

**Alekseevskiĭ, V. P. On the field of velocities in an infinite layer of an ideal incompressible liquid, induced by the action of instantaneous forces.** Ukrain. Mat. Ž. **11** (1959), 199–203. (Russian)

2205:

**Rouse, Hunter. Distribution of energy in regions of separation.** Houille Blanche **15** (1960), 235–246; French text, 221–234.

Author's summary: "During the past decade the Iowa Institute of Hydraulic Research has undertaken a series of studies of flow in the vicinity of quasi-stable eddies produced by separation of boundary angularities.

"The author describes in the present paper the determination of the mean and secondary patterns of axisymmetric flow for two comparable boundary forms: the abrupt inlet and the blunt shaft. Measurements available for analysis included the distributions of mean velocity, mean pressure, longitudinal and radial intensities of turbulence, turbulent shear, and longitudinal intensity gradient. Through use of the equations of momentum and of energy for the mean and the secondary motion, the measured distributions are adjusted to yield the required balance of the essential terms in the equations, thus yielding results in general accord with physical requirements. These are presented in the form of the flow

patterns themselves, supplemented by curves of variation of the individual momentum and energy terms throughout the regions of separation.

"Of particular significance is the interrelationship of turbulence production, convection, and dissipation, and the fact that a relatively small amount of turbulence energy is required locally to produce a large overall change in flow pattern.

"Two final diagrams indicate the variation of the terms of the Bernoulli equation extended to include the effects of turbulence and energy dissipation."

2206:

**Kito, Fumiki. On virtual mass of water contained in a rectangular tank whose side-walls are vibrating.** Proc. Fac. Engrg. Keio Univ. **11** (1958), no. 40, 1–19.

2207:

**Muggia, Aldo. Sulla teoria dei profili alari e delle schiere di profili alari.** Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **93** (1958/59), 301–312.

2208:

**Roy, Durga. Resistance on a circular cylinder due to any number of vortices lying in two rows.** Z. Angew. Math. Phys. **10** (1959), 502–508. (German summary)

The force on a circular cylinder in a uniform field of flow in which there are $n$ vortex pairs forming a wake is calculated using an extension of the Blasius formula to non-steady flow. The force per unit length is expressed as a double sum which is in a fairly simple form if the vortices are equally spaced in two rows parallel to the direction of the main flow. *W. R. Dean* (London)

2209:

**Abbott, Ira H.; von Doenhoff, Albert E. ★Theory of wing sections: Including a summary of airfoil data.** Dover Publications, Inc., New York, 1959. x+693 pp. Paperbound: $2.95.

This useful compilation first appeared in 1949. It begins with a concise review of the mathematical techniques of steady plane flow of an incompressible flow, and then presents both classical conformal-mapping methods and thin-airfoil theory. This latter is basically due to such pioneers as Munk, Birnbaum, and Glauert, but was systematized and improved by the present authors and their associates during World War II. They used it effectively to design the so-called low-drag airfoil profiles, whose pressure distributions are such as to maintain laminar boundary-layer flow over much of their surfaces. An admirable chapter on boundary-layer theory and profile drag is included here; the final chapter is concerned with compressibility effects and corrections at subsonic speeds. Finally, in appendices, half the book is devoted to tabulations of the computed characteristics of the N.A.C.A. airfoil families. 160 references are listed. *W. R. Sears* (Ithaca, N.Y.)

2210:

**Woods, L. C. On the theory of a cascade of stalled aerofoils.** J. Austral. Math. Soc. **1** (1959/61), 210–219.

The author shows how it is possible to evaluate the magnitude ($U$) and direction ($\beta$) of an inviscid, incompressible fluid after it has passed through a cascade of aerofoils. It is assumed that there is a stagnation point on the upper surface of each aerofoil and in this respect the paper is an extension of previous work by the same author. Expressions are given for the lift and drag of one aerofoil of the cascade in terms of $U$, $\beta$ and the initial flow conditions. The motion is taken to be two-dimensional so that the method of conformal transformations can be used. Several special cases are considered in detail; these include flow past or through a slotted wall and flow through a series of tubes. *G. N. Lance* (Winfrith, Dorset)

2211:

**Hancock, G. J. Notes on thin wing theory at low supersonic speeds. II. A unified numerical method for the calculation of the loading on wings with subsonic leading edges.** Aero. Quart. **10** (1959), 319–325.

A numerical procedure, based on the method described in the author's previous paper [Aero. Quart. **10** (1959), 247–265; MR **21** #7668], is described and applied to a delta wing. *J. W. Miles* (Los Angeles, Calif.)

2212:

**Jones, J. G. The pressure distribution of a wing with a subsonic leading edge in the presence of anti-symmetric body shaping.** Aero. Quart. **11** (1960), 51–70.

The paper is concerned only with the pressure distribution on a wing, which has a subsonic leading edge over at least part of the span. The wing is assumed to be centrally mounted on a quasi-cylindrical body; linearized theory is used so that the effects of anti-symmetric (with respect to wing plane) body distortions are independent of effects due to wing thickness, camber and incidence, body incidence and symmetric body distortions. Thus it is sufficient to consider a flat plate wing, at zero incidence attached to such a body. Briefly, Evvard's method is used and the body shape is represented by sources and multiples along the body axis.

*G. N. Lance* (Winfrith, Dorset)

2213:

**Honda, M. Theory of a thin wing in a shear flow.** Proc. Roy. Soc. London Ser. A **254** (1960), 372–394.

The problem attacked is that of a thin rectangular wing spanning an inviscid, incompressible shear flow between parallel plane walls. The small-perturbation approximation is made; i.e., the velocity perturbations due to the wing are supposed to be small compared to the stream speed $U(y)$, but $U'(y)$ need not be small. A linear second-order differential equation is derived for the pressure; this is reminiscent of the one derived by Lighthill [Quart. J. Mech. Appl. Math. **3** (1950), 303–325; MR **12**, 454] and others for a two-dimensional compressible flow. The solution is obtained in series form, involving a set of orthogonal functions of $y$ and rather complicated functions of $x$ and $z$. The lift and drag are calculated as sums over the coefficients of this series. The relationship to lifting-line theory is shown. Detailed calculations are made for a certain trapezoidal velocity profile $U(y)$, and the results include chordwise and spanwise lift distribution, pressure distribution on a bi-convex airfoil, induced

drag, and induced velocity in the far wake. In the case studied, where $U(y)$ drops at the walls to half its midstream value, there is a sharp increase of local lift coefficient toward the walls.

In two appendices the author treats a certain homogeneous integral equation with a symmetrical kernel and derives some integral relations involving Mathieu functions. In a third appendix he presents tabular values of some Fourier coefficients used in the preceding calculations.

*W. R. Sears* (Ithaca, N.Y.)

2214:

**Nishiyama, Tetsuo. Air-drawing and flow separation characteristics of a shallowly submerged hydrofoil section.** J. Zosen Kiokai **106** (1960), 9–17. (Japanese. English summary)

2215:

**Bessho, Masatoshi. On the theory of the wave resistance. II.** J. Zosen Kiokai **106** (1960), 1–7. (Japanese. English summary)

From the author's summary: "Following the theory of the first report [same J. **105** (1959), 1–6; MR **22** #428], the author tries to obtain the second approximation of the velocity potential of an infinite draft ship with parabolic waterline."

2216:

**Haskind, M. D. The radiation and diffraction of surface waves from a vertically floating plate.** Prikl. Mat. Meh. **23** (1959), 546–556 (Russian); translated as J. Appl. Math. Mech. **23**, 770–783.

The linearized plane problem is treated. The partially immersed vertical plate has a prescribed motion of rotation and of horizontal translation, and a regular wave train is incident from infinity. The solution uses the complex variable technique published by the author in 1948 [Akad. Nauk. SSSR Inž. Sb. **4** (1948), no. 2, 147–160; MR **12**, 762]. If $w$ denotes the complex velocity potential, then a combination of the form $Kw+idw/dz$ is pure imaginary on the mean free surface; also the boundary condition on the plate can be written as a boundary condition on the imaginary part of $Kw+idw/dz$. Thus $Kw+idw/dz$ can be found, and thence $w$ by solving an ordinary differential equation. (Similar techniques, with important extensions, were developed in the United States; see J. J. Stoker, *Water waves* [Interscience Publishers, Inc., New York, 1957; MR **21** #2438]. For a different treatment by integral equations see the reviewer's papers, Proc. Cambridge Philos. Soc. **43** (1947), 374–382; Quart. J. Mech. Appl. Math. **1** (1948), 246–252 [MR **9**, 117; **10**, 165].)

In the present paper the technique is employed to give the forces on the plate, and virtual-mass and damping coefficients in closed form in terms of Bessel and related functions. Some second-order-mean values are deduced.

*F. Ursell* (Cambridge, England)

2217:

**Bunimovič, A. I. Flow of a viscous rarefied gas around a plane semi-infinite plate.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 5, 16–18. (Russian)

2218:

**Il'čenko, V. I. Critical Reynolds number for flow behind a circular cylinder.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 5, 130–133. (Russian)

2219:

**Šašin, V. M. Viscous gas flow in the space between eccentric cylinders.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 179–182. (Russian)

Isothermal flow is considered in the thin gap between two fixed circular cylinders of finite length, gas entering under pressure through a number of circular orifices in the periphery. The usual Reynolds equation of lubrication theory is reduced, by expansion in powers of the eccentricity, to the Cauchy-Riemann equations for the basic term, followed by corresponding non-homogeneous systems for the higher approximations. The first two terms in the solution are found explicitly using complex variable theory.

*M. D. Van Dyke* (Stanford, Calif.)

2220:

**Toomre, Alar. The viscous secondary flow ahead of an infinite cylinder in a uniform parallel shear flow.** J. Fluid Mech. **7** (1960), 145–155.

It is supposed that the undisturbed flow upstream of the cylinder is weakly sheared, its vorticity being perpendicular to the axis of the cylinder. First the author gives a simple method for obtaining the secondary inviscid flow induced by the cylinder. He then goes on to calculate its effect on the boundary layer near the stagnation line, which is complicated by a logarithmic singularity in the secondary flow at the body. Finally, the change in the total pressure of a fluid particle is calculated as it moves from infinity upstream to the stagnation line.

*K. Stewartson* (Durham)

2221:

**Sparrow, E. M.; Eichhorn, R.; Gregg, J. L. Combined forced and free convection in a boundary layer flow.** Phys. Fluids **2** (1959), 319–328.

This paper is concerned with the boundary layer flow and temperature layer around a heated or cooled body and created by the simultaneous action of forced and free convection. Under the assumption that the flow is laminar and steady and that viscous dissipation can be neglected, the authors investigate the similar solutions of the boundary layer equations. For a free stream velocity $U_\infty = Ax^m$ and a wall temperature $T_w = T_\infty + Bx^n$ ($T_\infty$ the constant free-stream temperature) it is found that similar solutions exist only if $n = 2m-1$. The resulting system of ordinary differential equations is solved numerically for the two cases: constant wall temperature and constant wall heat flux, and these numerical results are discussed in detail.

*W. C. Rheinboldt* (Syracuse, N.Y.)

2222:

**Sinbel, M. A. Boundary-layer solution for a flow in a diverging passage having a swirl component.** J. Appl. Mech. **26** (1959), 477–484.

In steady viscous flow the velocity components $(u, v, w)$ referred to spherical polar coordinates $(r, \theta, \phi)$ are assumed

to be independent of $\phi$, and a boundary layer near the fixed cone $\theta=\alpha$ is considered. After integration of the equations through the thickness $\delta$ of the boundary layer, $v$ is eliminated and $u$, $w$ are expressed following the Pohlhausen method as polynomials of degree 4. For turbulent motion $\partial u/\partial\theta$ is replaced by $\partial u/\partial\theta+(l\partial u/\partial\theta)^2/(\nu\delta)$. The solution for laminar flow is compared with two others by A. M. Binnie and D. P. Harris [Quart. J. Mech. Appl. Math. **3** (1950), 89–106; MR **11**, 698] and G. I. Taylor [ibid. **3** (1950), 129–139; MR **11**, 697]. Fair qualitative agreement is shown between the results of the analysis and those of some experiments which are described.

*W. R. Dean* (London)

2223a:

**v. Krzywoblocki, M. Z. On the generalized theory of the laminar, two-dimensional boundary layer along a flat plate in continuum and slip flow regimes. I, II.** Bull. Soc. Math. Grèce **29**, 34–73 (1954).

2223b:

**v. Krzywoblocki, M. Z. On the fundamental system of equations of the laminar, two-dimensional boundary layer along a flat plate in continuum and slip flow regimes.** Bull. Soc. Math. Grèce **31**, 41–68 (1959).

The work of A. J. A. Morgan [Quart. J. Math. Oxford Ser. (2) **3** (1952), 250–259; MR **15**, 37] on the relations between certain ordinary and partial differential equations are generalised and combined with that of Carathéodory on the fundamental theory of differential systems containing a parameter.

The results are used to derive the boundary-layer equations pertinent to flow along a flat plate and to Grad's equations.

*K. Stewartson* (Durham)

2224:

**Roy, Ajit Kumar. Supersonic viscous flow past a convex corner.** Z. Angew. Math. Phys. **10** (1959), 592–603. (German summary)

The author aims to take account of the "rounding off" effect of the laminar boundary layer on the Prandtl-Meyer flow past a convex corner. He appears to assume that the wall streamline (with corner) is replaced by a smooth streamline of the inviscid Prandtl-Meyer family and then employs Polhausen methods to determine the flow.

*H. C. Levey* (Nedlands, Australia)

2225:

**Četaev, N. G. On the stability of the rotational motions of a hollow body filled with an ideal liquid.** Prikl. Mat. Meh. **21** (1957), 157–168. (Russian)

2226:

**Nakagawa, Yoshinari. Heat transport by convection.** Phys. Fluids **3** (1960), 82–86.

The amplitude of the cellular motion near a convective instability is approximated by J. T. Stuart's integral method [J. Fluid Mech. **4** (1958), 1–21; MR **19**, 1221]. The analysis and results are identical with those found in the author's references.

*W. V. R. Malkus* (Woods Hole, Mass.)

2227:

**Nakagawa, Yoshinari. Heat transport by convection in presence of an impressed magnetic field.** Phys. Fluids **3** (1960), 87–93.

Continuation of #2226. For rigid boundary conditions, the amplitude is reduced by an imposed magnetic field.

*W. V. R. Malkus* (Woods Hole, Mass.)

2228:

**Case, K. M. Stability of an idealized atmosphere. I. Discussion of results.** Phys. Fluids **3** (1960), 149–154.

Consider the uniform shear flow of an ideal, incompressible fluid with velocity $Ry$ and density $A\exp(-\beta y)$ above a horizontal plane $y=0$ with $Q=\beta g/R^2$ as the Richardson number. G. I. Taylor [Proc. Roy. Soc. London Ser. A **132** (1931), 499–523], neglecting the kinematic effects of density variation, as compared with buoyancy effects, concluded that if $Q>\frac{1}{4}$ "progressive waves can exist in the medium, but if $[0<Q<\frac{1}{4}]$ no waves either progressive or exponentially unstable can exist". The author, referring to Taylor's analysis only as "somewhat approximate and incomplete" and to Richardson not at all, attributes not only the foregoing result, but also the very concept of the Richardson number, to Dyson (unpublished). He then proceeds to obtain an asymptotic solution to the initial value problem (Taylor and Dyson having considered only harmonic motions) and concludes that an arbitrary disturbance decays as $t^{-a}$, $a=\frac{1}{2}-(\frac{1}{4}-Q)^{1/2}$, $Q>0$. He appears to have been unaware of the earlier work of Eliassen, Høiland and Riss [Institute for Weather and Climate Research, the Norwegian Academy of Sciences and Letters, Publ. No. 1, 1953; MR **16**, 642], who obtained this same result for $Q>-2$ on the basis of Taylor's approximation with respect to density variation and showed that it applies also to flow between parallel planes for $Q>-\frac{3}{4}$.

*J. W. Miles* (Los Angeles, Calif.)

2229:

**Dyson, Freeman J. Stability of an idealized atmosphere. II. Zeros of the confluent hypergeometric function.** Phys. Fluids **3** (1960), 155–157.

The author, supporting the preceding analysis by Case [see preceding review], extends Tricomi's [Math. Z. **52** (1950), 669–675; MR **12**, 256] enumeration of the zeros of the Whittaker function $W_{k,m}(z)$ for real $k$ and $m$. He proves that when $k$ is real and $m$ is pure imaginary $W_{k,m}(z)$ has no complex zeros and has an infinite set of positive real zeros with a point of accumulation at zero.

*J. W. Miles* (Los Angeles, Calif.)

2230:

**Kolomý, Josef. An application of Galerkin's method to stability problems of a stream line viscous flow.** Apl. Mat. **5** (1960), 40–44. (Czech. Russian and English summaries)

The purpose of the note is to consider the applicability of Galerkin's method to the stability problems. In particular, the author attacks the problem of the calculation of eigenvalues of the stability equation of a viscous flow between two coaxial infinite cylinders rotating in the same or opposite directions. The author works in an $m$-dimensional Euclidean space, introduces the Green's function and some classical operators, and using the

classical limiting process proves the convergence of Galerkin's method for the calculation of eigenvalues in the problem in question.

*M. Z. v. Krzywoblocki* (E. Lansing, Mich.)

2231:

**Ludwieg, Hubert. Stabilität der Strömung in einem zylindrischen Ringraum.** Z. Flugwiss. 8 (1960), 135–140. (English and French summaries)

Author's summary: "The present paper deals with the investigation of the helical flow in an annulus between two coaxial cylinders as regards its stability against the formation of helical vortices of the type known as Taylor's annular vortices. Assuming the annulus to be small and the velocities to vary linearly with the radius, it is shown that the problem can be reduced to the classical case of flow between two rotating cylinders. An appropriate stability criterion for helical flows is derived from Rayleigh's stability criterion applicable to such flows."

2232:

**Pantchev, Stoitcho. Sur la théorie statistique de la turbulence.** C. R. Acad. Sci. Paris **250** (1960), 661–662.

Par une hypothèse de quasi-normalité, Chandrasekhar a établi [Proc. Roy. Soc. London Ser. A **210** (1951), 18–25; **229** (1955), 1–19; MR **13**, 596; **16**, 968] une équation aux dérivées partielles pour les corrélations de vitesse dans un fluide turbulent homogène et stationnaire. L'auteur introduit l'équation de propagation de la température et établit avec les mêmes hypothèses une équation aux dérivées partielles qui permet en principe de calculer les corrélations de température lorsque les corrélations de vitesse sont déjà déterminées.

*J. Bass* (Paris)

2233:

**Ginevskiĭ, A. S.; Solodkin, E. E. The effect of lateral surface curvature on the characteristics of axially-symmetric turbulent boundary layers.** J. Appl. Math. Mech. **22** (1958), 1169–1179 (819–825 Prikl. Mat. Meh.).

The equation of motion in the turbulent boundary layer on an axi-symmetric surface is solved by equating the first three terms of a Taylor expansion for the shear stress to the value $\rho l^2(\partial u/\partial y)^2$, with $l=ky$, given by mixing length theory, in the turbulent part of the layer and to $\mu\partial u/\partial y$ in the sublayer. In this way closed expressions for the velocity profile and the skin friction coefficient are found. These show that the velocity profile is "fuller", i.e., rises more rapidly near the wall, on a convex surface, and less full on a concave surface, than on a flat surface with the same longitudinal pressure gradient $dp/dx$.

The mixing length expression is adopted somewhat uncritically, and its use strikes this reviewer as highly speculative. In view of the well-known "law of the wall" one would expect there to be some relation between $l$ and $dp/dx$ even on a plane surface, and in the general case might not $l$ depend on the wall curvature as well? Moreover the Taylor expansion of the product $\tau\tau$ excluding terms of order $y^3$ cannot be expected to be accurate far from the wall, and certainly does not show zero shear stress at the outer edge where a boundary condition on the velocity profile is applied. But despite these reservations it must be conceded that the agreement with experimental values of skin friction on cylinders at zero pressure gradient is very satisfactory.

*D. A. Spence* (Pasadena, Calif.)

2234:

**Helliwell, J. B.; Mackie, A. G. The flow past a closed body in a high subsonic stream.** Quart. J. Mech. Appl. Math. **12** (1959), 298–313.

Le problème étudié est celui de l'écoulement autour d'un profil symétrique à la portance nulle dans le cadre de la théorie des petites perturbations transsoniques. Le profil est très particulier: l'extrados comprend deux segments $BC$ et $EF$ symétriques par rapport à la médiatrice de $BC$—$B$ est le nez et $F$ la pointe arrière—reliés par un arc continu $CDE$ dessiné pour que la vitesse reste constamment sonique le long de cet arc.

Dans le plan de l'hodographe, il s'agit donc de résoudre un problème aux limites pour l'équation de Tricomi. Après avoir formé une solution présentant la singularité voulue au point image de l'écoulement à l'infini, les auteurs parviennent, après d'ingénieuses mais délicates transformations, à satisfaire les conditions aux limites en résolvant de façon approchée une équation intégrale. Une étude complète des séries permettant la détermination géométrique du profil conduit à des applications numériques précises de ce travail théorique.

*P. Germain* (Paris)

2235:

**Ryžov, O. S. Spatial transonic gas flows in ducts.** Prikl. Mat. Meh. **23** (1959), 781–784 (Russian); translated as J. Appl. Math. Mech. **23**, 1115–1121.

Des solutions de l'équation aux dérivées partielles régissant les écoulements transsoniques voisins d'un écoulement uniforme sont construites en vue d'obtenir des écoulements tridimensionnels non nécessairement de révolution dans une tuyère.

Par un choix convenable des constantes, on peut ainsi tracer des écoulements ayant des vitesses supersoniques dans certaines régions voisines du col de la tuyère. Ce travail généralise les résultats donnés par S. Tomotika et K. Tamada [Quart. Appl. Math. **7** (1950), 381–397; MR **11**, 275] dans le cas des écoulements plans.

*P. Germain* (Paris)

2236:

**Nocilla, Silvio. Flussi transonici attorno a profili alari simmetrici con onda d'urto attaccata ($M_\infty<1$).** Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **92** (1957/58), 282–307.

The author investigates a symmetric transonic flow, with $M_\infty<1$, around a profile assuming that there exists a single shock which begins at the body and ends within the supersonic region, and not on the sonic line as in the model proposed by Frankl [Prikl. Mat. Meh. **19** (1955), 385–392; **20** (1956), 196–202; MR **17**, 550; **18**, 255; cf. Germain, C. R. Acad. Sci. Paris **243** (1956), 1190–1192; MR **18**, 695]. The study is local: the author is concerned only with the behavior of the flow near the point $J$ where the shock is transformed into a compression characteristic. He formulates the conditions which must be satisfied by a solution (stream function) $\psi$ in the hodograph plane near the point $J'$ corresponding to $J$. Assuming a Tricomi gas, he constructs a "first approximation" to the solution. *C. S. Morawetz* (New York)

2237:

**Rogers, E. W. E.; Hall, I. M. An introduction to the flow about plane swept-back wings at transonic speeds.** J. Roy. Aero. Soc. **64** (1960), 449–464.

Authors' summary: "A brief survey is made of the way the flow develops about wings with leading edges swept at about 50° as the stream Mach number rises from a subsonic to a supersonic value. The shock pattern which occurs may be complex. Three aspects of the wing flow are discussed in more detail, including the possible conditions for shock-induced separation of the surface boundary layer. The effect of the changing flow pattern on the overall wing lift, drag and pitching moment is commented upon."

2238:

**Želudev, P. I. On supersonic flow past cylindrical plumed bodies of revolution.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 5, 118–121. (Russian)

The author attempts to iterate on slender-body theory to find a second approximation for the lift of narrow flat wings at the cylindrical base of a body. However, the essential square-root singularity at the leading edge is obliterated by an unjustified series expansion.

*M. D. Van Dyke* (Stanford, Calif.)

2239:

**Harrison, William P., Jr. Computer solution techniques for two-dimensional supersonic flow problems.** J. Aero/Space Sci. **27** (1960), 315–316.

2240:

**Charwat, Andrew F. A survey of hypersonic problems and the characteristics of shock-heated tunnels.** Z. Flugwiss. **8** (1960), 125–134. (German and French summaries)

Author's summary: "In the first part of this paper, the characteristic phenomena of hypersonic aerodynamics are discussed and outlined in the domain of Mach and Reynolds number. Regions of similarity and interactions between these phenomena are schematically defined and used to evaluate the range of application of experimental facilities, in particular, of the hypersonic shock-tunnel. In the second part, the principle of operation, the problems and the limitations of shock-heated tunnels are reviewed."

2241:

**Holt, Maurice. A linear perturbation method for stability and flutter calculations on hypersonic bodies.** J. Aero/Space Sci. **26** (1959), 787–793.

A numerical scheme is developed for calculating infinitesimal time-dependent perturbations of a basic steady supersonic flow that has itself been computed by the numerical method of characteristics. Potential flow is assumed for simplicity. As an example, the stability derivatives are calculated for a conical boat-tail at Mach number 3.05. *M. D. Van Dyke* (Stanford, Calif.)

2242:

**Talbot, L.; Koga, T.; Sherman, P. M. Hypersonic viscous flow over slender cones.** J. Aero/Space Sci. **26** (1959), 723–730.

Pressure distributions were measured on unyawed circular cones of 3° semi-vertex angle at Mach numbers from 3.7 to 5.7 and values of the viscous interaction parameter from 0.5 to 2.3. The pressure increase induced by the boundary layer was estimated with good accuracy using the tangent-cone method together with an approximate formula for the displacement thickness.

*M. D. Van Dyke* (Stanford, Calif.)

2243:

**Hayes, Wallace D.; Probstein, Ronald F. Viscous hypersonic similitude.** J. Aero/Space Sci. **26** (1959), 815–824.

A careful and comprehensive study is made of the conditions under which the hypersonic similarity rule can be generalized to include the displacement effect of the laminar boundary layer. Three forms of similitude are found, each requiring additional assumptions regarding the properties of the gas. Special discussion is devoted to three-dimensional effects, rarefied-gas effects, the limit of weak interaction, and blunt noses.

*M. D. Van Dyke* (Stanford, Calif.)

2244:

**Luniev, V. V. On the similarity of hypersonic viscous flows around slender bodies.** J. Appl. Math. Mech. **23** (1959), 273–280 (193–197 Prikl. Mat. Meh.).

One of Hayes and Probstein's three forms of the viscous hypersonic similarity rule [see preceding review] is deduced. *M. D. Van Dyke* (Stanford, Calif.)

2245:

**Whalen, Robert J. Viscous hypersonic similitude for a dissociating gas.** J. Aero/Space Sci. **27** (1960), 550–551.

2246:

**Pottsepp, L. Inviscid hypersonic flow over unyawed circular cones.** J. Aero/Space Sci. **27** (1960), 558–559.

2247:

**Cherny, G. G. Hypersonic flow past an aerofoil with a slightly blunted leading edge.** Dokl. Akad. Nauk SSSR (N.S.) **114** (1957), 721–724. (Russian)

According to the unsteady analogy, steady flow past a blunt airfoil corresponds to one-dimensional piston motion starting with instantaneous release of energy and momentum. This problem is solved approximately assuming that the gas is concentrated just behind the shock wave. Thus the pressure distribution and shock wave shape are calculated for a slightly blunted wedge in hypersonic flow. *M. D. Van Dyke* (Stanford, Calif.)

2248:

**Lyubimov, G. A. High supersonic non-ideal gas flow around bodies.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 173–176. (Russian)

Černyĭ's approximate method of solving hypersonic flow past plane or axisymmetric bodies by expanding in

powers of $(\gamma-1)/(\gamma+1)$, where $\gamma$ is the adiabatic exponent [Dokl. Akad. Nauk SSSR **107** (1956), 221–224; MR 18, 167], is generalized to non-ideal gases. As an example, results of the first approximation are shown for a circular cone in air, the thermodynamic properties having been taken from existing tables.

*M. D. Van Dyke* (Stanford, Calif.)

2249:

**Gonor, A. L. High supersonic gas flow around conical bodies.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 34–40. (Russian)

Conical flows are treated by the method, introduced by Černyĭ for plane and axisymmetric hypersonic flow [Dokl. Akad. Nauk SSSR **107** (1956), 221–224; MR 18, 167] of expanding in powers of $(\gamma-1)/(\gamma+1)$, where $\gamma$ is the adiabatic exponent. The second approximation is calculated for an inclined flat triangular wing with supersonic edges, and the first approximation for an unyawed elliptic cone. *M. D. Van Dyke* (Stanford, Calif.)

2250:

**Lyahov, G. M. Shock waves in multi-component media.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 1, 46–49. (Russian)

2251:

**Van Tuyl, A. H. The use of rational approximations in the calculation of flows with detached shocks.** J. Aero/Space Sci. **27** (1960), 559–560.

2252:

**Powell, Alan. Propagation of a pressure pulse in a compressible flow.** J. Acoust. Soc. Amer. **31** (1959), 1527–1535.

A one-dimensional treatment is given of the propagation of a small pressure pulse through a duct of variable cross-section carrying a compressible flow. The analysis ignores the governing partial differential equation and develops series for the reflection and transmission coefficients of the duct in terms of the entry and exit mach numbers. The successive terms of the series account for multiple reflections of increasing order within the variable area portion of the duct. *A. F. Pillow* (Toronto)

2253:

**Teipel, Ingolf. Starke kugelsymmetrische Verdichtungsstösse unter Berücksichtigung von Dissoziations- und Ionisationsvorgängen.** Z. Flugwiss. **8** (1960), 187–202. (English and French summaries)

Author's summary: "Starting from the fundamental equations of gas dynamics, a converging spherical shock wave was calculated, including effects of dissociation and ionization. Behind the shock front the flow is assumed to be quasi-steady, an assumption which gives the exact solution at high pressures for ideal gases of constant specific heats. Hydrogen is used as a medium. The calculation has been carried out step by step till $T=10{,}000°K$. It is shown that the gas is nearly completely dissociated, before the ionization begins. The equations become very simple, when the gas behind the shock front has been ionized."

2254:

**Devienne, F. M. (Editor). ★Rarefied gas dynamics: Proceedings of the First International Symposium held at Nice.** International Series on Aeronautical Sciences and Space Flight, Division IX, Vol. 3. Pergamon Press, New York-London-Oxford-Paris, 1960. vii+442 pp. (20 plates) $17.50.

These 29 papers range from comprehensive survey articles to brief notes. Notable in the first category are Sherman and Talbot's review of "Experiment versus kinetic theory for rarefied gases" based on three standard problems, summaries of methods of solving the Boltzmann equation by Grad and by Gross, and Stalder's survey of the use of low-density wind tunnels. Several more specific papers are concerned with improving on free-molecule theory by successive approximations. The remainder, mainly theoretical, range as far afield as continuum magnetohydrodynamics.

*M. D. Van Dyke* (Stanford, Calif.)

2255:

**Baker, R. M. L., Jr.; Charwat, A. F. Transitional correction to the drag of a sphere in free molecule flow.** Phys. Fluids **1** (1958), 73–81.

Authors' summary: "The mechanics of collisions between molecules in front of a sphere moving at hypervelocity through a rarefied atmosphere are analyzed. The study is concerned with the 'transitional' regime in which molecules emitted by the surface begin to shield the body from the oncoming Newtonian stream and the net drag decreases from its free-molecular value. It is found that a simplified but physically significant model can be formulated, and that it leads to a dependence of the drag coefficient on two transition parameters. These can be interpreted as the Reynolds number and the surface-to-free stream temperature ratio. The predictions of this theory agree qualitatively and in order of magnitude with observations."

*C. D. Calsoyas* (Livermore, Calif.)

2256:

**Urusovskiĭ, I. A. Sound scattering by a sinusoidally uneven surface characterized by normal acoustic conductivity.** Akust. Ž. **5** (1959), 355–362 (Russian); translated as Soviet Physics. Acoust. **5** (1960), 362–369.

Approximate relations are established for the distribution of acoustic pressure along an uneven scattering surface of sufficiently small slope with varying normal acoustic admittance. The field above the surface is determined in terms of that at the surface by Green's formula. In particular, the case of a plane wave incident at an arbitrary angle on a sinusoidal surface with sinusoidally-varying admittance is considered. The results clarify limitations of the Rayleigh and Kirchhoff methods for scattering from periodic surfaces.

*W. W. Soroka* (Berkeley, Calif.)

2257:

**Nesterov, V. S. Viscous-inertial dispersion and attenuation of sound in a highly concentrated suspension.** Akust. Ž. **5** (1959), 337–344 (Russian); translated as Soviet Physics. Acoust. **5** (1960), 344–351.

The simple theory of sound propagation in suspensions as formulated by Lamb does not take into account the interaction of particles and in this paper the problem is

treated in terms of a model consisting of a system of small cylinders whose axes lie along the direction of propagation with a small quantity of viscous liquid filling the interstices between them. In this model, for high concentrations, it is the viscosity of the liquid and the inertia of the cylinders which are the relevant parameters. It is shown that this model accounts for the observed behaviour of concentrated suspension qualitatively and to some extent quantitatively. It is suggested that the results should be of importance in architectural acoustics and in subterranean hydraulics. *H. Kolsky* (London)

2258:

**Millar, R. F. A note on diffraction by an infinite slit.** Canad. J. Phys. **38** (1960), 38–47.

The two-dimensional problem of diffraction of a scalar monochromatic plane wave by a narrow slit in an infinite screen is considered for the case of oblique incidence. The total wave function or its normal derivative vanishes at the screen, and accordingly a differential-integral equation or an integral equation is set up for the unknown function in the slit. These equations are solved in a series of powers of $\varepsilon$ and $\log \varepsilon$, where $\varepsilon/\pi$ is the ratio of slit width to wavelength. Expressions are found for the transmission coefficients as functions of $\varepsilon$ and the angle of incidence. The author's work slightly extends that of A. T. de Hoop [Nederl. Akad. Wetensch. Proc. Ser. B **58** (1955), 401–411; MR **17**, 559]. *C. J. Bouwkamp* (Eindhoven)

2259:

**Young, F. J. The natural frequencies of musical horns.** Acustica **10** (1960), 91–97. (German and French summaries)

Author's summary: "A general method for the precise calculation of the natural frequencies of horns having any cross-sectional area versus length characteristic is devised. The new method is applied to the Flugelhorn and the tenor trombone. The results are critically examined with respect to various musical scales and the role of the internal impedance of the musician is discussed. Means by which the intonation of horns might be improved are presented."

2260:

**Heckl, M. Untersuchungen an orthotropen Platten.** Acustica **10** (1960), 109–115. (English and French summaries)

Author's summary: "Starting from the equations of Huber for orthotropic sheets, the flexural wave input impedance, the mean particle velocity and the radiated sound power are calculated for the case of excitation at one point. Furthermore the sound attenuation of infinitely extended sheets is calculated. The expressions derived are checked by measurements with corrugated and slotted sheets."

2261:

**Pticyn, O. B.; Ĕĭzner, Yu. E. Hydrodynamics of polymer solutions. II. Hydrodynamic properties of macromolecules in active solvents.** Ž. Tehn. Fiz. **29** (1959), 1117–1134 (Russian); translated as Soviet Physics. Tech. Phys. **4** (1960), 1020–1036.

Part I [same Ž. **29** (1959), 75–93] is listed as MR **21** #5396.

2262:

**Mithal, K. G. Motion of a non-Newtonian fluid produced by the uniform rotation of a plate.** Ganita **9** (1958), 95–117.

The author deals with a special kind of incompressible Reiner-Rivlin fluid. He investigates the flow produced by a rotating infinite plate immersed in a fluid filling all space. The method is analogous to the one used by von Kármán [Z. Angew. Math. Mech. **1** (1921), 233–252] for Newtonian fluids. *W. Noll* (Pittsburgh, Pa.)

2263:

**Fraenkel, L. E. A shallow-liquid theory in magnetohydrodynamics.** J. Fluid Mech. **7** (1960), 81–107.

This paper considers the extension of the nonlinear and linear shallow water theories to the case of an electrically conducting fluid in the presence of a vertical magnetic field. In the first of three parts, the basic approximations are introduced and the equations and boundary conditions governing the motion are derived. Explicit solutions of the linear boundary value problem are presented in the second section. The main conclusion is that the wave amplitudes in a disk and the amplitudes behind progressing fronts are exponentially damped. The mechanical energy is dissipated by Joule heating. The "disappearance" of waves under the action of a magnetic field agrees with experimental results. Although previous explanations suggested an induced effective viscosity, this theory involves a resisting force which is proportional to the velocity (and not to the second derivatives). The third section concerns the nonlinear theory and the approach is based on the method of characteristics. The tendency of waves to break or form bores is inhibited by the presence of a magnetic field. Waves which ordinarily break, such as those propagating into quiescent water with a finite slope, need not do so in the present circumstances. *H. Greenspan* (Cambridge, Mass.)

2264:

**Gupta, Anadi Shankar. Flow of an electrically conducting fluid past a porous flat plate in the presence of a transverse magnetic field.** Z. Angew. Math. Phys. **11** (1960), 43–50. (German summary)

If the magnetic field $\mathbf{B}$ and velocity $\mathbf{q}$ in an incompressible fluid of density $\rho$ depend only on the Cartesian coordinate $y$ and not on $x$, $z$, or time, then $B_y$ and $q_y$ are constant. If, moreover, $\mathbf{B}$ and $\mathbf{q}$ approach given finite limits $\mathbf{B}_\infty$ and $\mathbf{q}_\infty$ as $y \to \infty$, then the solutions $\mathbf{q}$, $\mathbf{B}$ of the hydromagnetic equations in $0 \leq y < \infty$ are: (i) unique if $c_y \leq q_y$; (ii) a one-parameter family if $-c_y \leq q_y < c_y$; (iii) a two-parameter family if $q_y < -c_y$. Here $c_y = (B_y^2/\mu_0\rho)^{1/2}$ is the Alfven speed in the $y$ direction. The author writes down the solutions in case (ii), and also finds the temperature distribution in the fluid. He erroneously claims that case (i) as well as case (iii) permits a two-parameter family of solutions, and dismisses both cases as "indeterminate".

*G. E. Backus* (La Jolla, Calif.)

2265:

**Talbot, Lawrence. Theory of the stagnation-point Langmuir probe.** Phys. Fluids **3** (1960), 289–298.

Author's summary: "A theory is developed for a Langmuir-type probe consisting of a collecting electrode placed at the stagnation point of a blunt body immersed in a supersonic partially ionized stream. It is shown that under certain conditions, the stagnation-point boundary layer equations and the probe sheath equations can be solved together to yield potential vs. current relationships which permit the free stream ion and electron densities and temperatures to be measured by such a probe. It is shown also that the stagnation-point heat transfer will vary with probe potential, thus providing additional information useful in plasma jet diagnostics."

2266:

**Kanwal, R. P. Uniqueness of magnetohydrodynamic flows.** Arch. Rational Mech. Anal. **4**, 335–340 (1960).

L'auteur établit un théorème d'unicité pour un problème aux limites portant sur les équations de la magnétodynamique des fluides. Les données sont relatives aux valeurs initiales de la vitesse, du champ magnétique, de le température et de la masse spécifique dans un domaine borné de l'espace, ainsi qu'aux valeurs, à tout instant, de certaines des grandeurs précédentes ou de leurs combinaisons, sur la frontière du domaine. Des inégalités sont établies et la méthode des fonctions majorantes utilisée. *H. Cabannes* (Paris)

2267:

**Stix, Thomas H. Generation and thermalization of plasma waves.** Phys. Fluids **1** (1958), 308–317.

Resistivity, gravity, viscosity and electron inertia are neglected. Standard methods lead for cylindrical symmetry to a dispersion relation which depends on the velocity distribution function. Four simple cases are considered: (I) neighborhood of ion cyclotron frequency; (II) ion cyclotron waves; (III) torsional hydromagnetic waves; (IV) compressional hydromagnetic waves. In the last three cases, the electric field in the plasma is in quadrature with the ion velocity. However, when these waves are externally generated by an induction coil, the electric field due to the coil appears in phase with the ion velocity, so that power can be transferred to the plasma. In case (II) the transfer can be very efficient. Thermalization appears when appreciable numbers of ions stream through the periodic perturbation with velocities such that, in their own rest frames, these ions "feel" the perturbation at their own cyclotron frequency.

Applications are considered for thermonuclear reactors. *E. Schatzman* (Paris)

2268:

**Kemp, Nelson H. Hydromagnetic effects on heating and shear at a three-dimensional stagnation point in hypersonic flow.** J. Aero/Space Sci. **27** (1960), 553–554.

2269:

**Kusukawa, Ken-ichi. On the high-speed flow of a compressible conductive fluid past a slender body.** J. Aero/Space Sci. **27** (1960), 551–553.

2270:

**Demetriades, A. A possible "fully developed" hydromagnetic pipe flow.** J. Aero/Space Sci. **27** (1960), 388–389.

2271:

**Percival, I. C. Waves in a conducting sheet situated in a strong magnetic field.** Proc. Phys. Soc. **76** (1960), 329–336.

Author's summary: "The hydromagnetic approximation is applied to the elementary linear theory of transverse waves in a thin uniform plane conducting sheet, in which the inertia is provided by the sheet, and the restoring forces by strong vacuum magnetic fields on either side of the sheet. The dispersion relation and damping are obtained. The waves should be observable in the laboratory."

2272:

**Freeman, N. C. On the flow past a sphere at hypersonic speed with a magnetic field.** J. Aero/Space Sci. **26** (1959), 670–672.

In adding a magnetic field to the constant-density solution, Kemp [same J. **25** (1958), 405–407; MR **19**, 1123] approximated the stream function by a series expansion from the shock wave. That expansion is shown to be invalid, and is corrected using Lighthill's technique of straining coordinates. The stand-off distance and surface pressure are then found to be independent of magnetic field to first order. *M. D. Van Dyke* (Stanford, Calif.)

2273:

**Kemp, Nelson H. Author's reply.** J. Aero/Space Sci. **26** (1959), 672.

The author agrees with Freeman's criticism [see preceding review], and gives results of numerical integrations that confirm it. *M. D. Van Dyke* (Stanford, Calif.)

2274:

**Harris, Edward G. Relativistic magnetohydrodynamics.** Phys. Rev. (2) **108** (1957), 1357–1360.

In order to understand why twice the energy density of the magnetic field appears to contribute to the inertia of a perfectly conducting fluid, the author formulates the equations of magnetohydrodynamics in relativistic form. A relativistically correct equation for the production of entropy due to Joule heating is derived from the conservation laws for mass, momentum and energy. It is shown that, for motions perpendicular to the field, both the energy density and pressure of the magnetic field contribute to the inertia in the same way as do the internal energy and pressure of the fluid, thus resolving the author's problem. [For a nonrelativistic investigation in which the same phenomenon is evident see Lüst, Z. Naturf. **8a** (1953), 277–284; MR **15**, 271; or Westfold, Phil. Mag. (8) **2** (1957), 1287–1302.] The general dispersion relation for small-amplitude oscillations is also found.

*K. C. Westfold* (Pasadena, Calif.)

2275:

**Frieman, Edward A.; Kulsrud, Russell M. Problems in hydromagnetics.** Advances in applied mechanics, Vol. V, pp. 195–231. Academic Press Inc., New York, N.Y., 1958. x+459 pp. $12.00.

This is a review article which is not intended to be comprehensive, but rather to complement those by Elsasser [Amer. J. Phys. **23** (1955), 590–609; **24** (1956), 85–110], Lundquist [Ark. Fys. **5** (1952), 297–347; MR **14**, 814] and Chandrasekhar [Monthly Not. Roy. Astr. Soc. **113** (1953), 667–678; MR **16**, 84], and the book by Spitzer [*Physics of fully ionized gases*, Interscience, New York, 1956]. First the motion of the magnetic field lines and the conservation of energy in a compressible fluid of infinite conductivity are considered. [For a parallel discussion of the case of an ionized gas see Westfold, Phil. Mag. (8) **2** (1957), 1287–1302.] A review of recent work on the stability of hydromagnetic equilibria by Bernstein, Kruskal and the authors [Proc. Roy. Soc. London Ser. A. **244** (1958), 17–40; MR **19**, 1009] is then given, followed by applications to the pinch effect and to a general axisymmetric system. Finally, the three modes of propagation of plane hydromagnetic waves are investigated and limiting cases briefly discussed. Energy and wave generation, group velocity, reflection and refraction, damping, and more general waves are considered in turn, the last by means of the energy principle used in the stability investigations. {Much of this has previously been investigated by Baños [Proc. Roy. Soc. London Ser. A. **233** (1955), 350–366; MR **17**, 921], whose work has apparently been overlooked by the authors.}

*K. O. Westfold* (Pasadena, Calif.)

2276:

**Frankl', F. I. On a possible system of equations of relativistic gas dynamics considering radiation and absorption of light.** Dokl. Akad. Nauk SSSR **127** (1959), 987–989 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 786–788.

2277:

**Goodknight, Richard C.; Klikoff, Waldemar A., Jr.; Fatt, Irving. Non-steady-state fluid flow and diffusion in porous media containing dead-end pore volume.** J. Phys. Chem. **64** (1960), 1162–1168.

Authors' summary: "A system of equations is derived that describes pressure transients during non-steady-state flow of a fluid through a linear porous medium which contains dead-end pore volume. Analogous equations are shown to describe non-steady-state diffusion through fluid contained in such porous media. The equations are solved for a set of boundary conditions which are identical to those for which laboratory data were obtained. The theoretical and experimental results are in excellent agreement, thereby establishing the validity of the equations."

2278:

**Roughton, F. J. W. Diffusion through membranes followed by diffusion with rapid irreversible immobilization in another medium.** Trans. Faraday Soc. **56** (1960), 1085–1094.

Author's summary: "Equations are developed for the penetration of a diffusible substance from a medium (I) through a membrane into a second medium (II) in which the substance not only diffuses but is also immobilized rapidly and irreversibly on a finite number of sites. The mathematical treatment makes use of quasi-steady state principles and is applied to the three cases of layers, cylinders and spheres. Equations are given both for a finite amount and for an unlimited amount of diffusible substance in medium I.

"Numerical examples are worked out for the layer, the cylinder and the sphere and in each case it is shown that if the membrane between medium I and II is very thin, the effect of the membrane on the time for a given percentage uptake in medium II is linearly related to the quotient of the permeability of medium II by the permeability of the membrane."

## OPTICS, ELECTROMAGNETIC THEORY, CIRCUITS

See also 1981, 2258, 2267.

2279:

**Günther, N. Studien zur Theorie des Raumsehens. I.** Optik **17** (1960), 90–97. (English and French summaries)

Author's summary: "By means of a simplified model of the eye, equations are deduced from which it is possible to calculate the perspective distance at which an object under observation is presented to the eye."

2280:

**Günther, N. Studien zur Theorie des Raumsehens. II.** Optik **17** (1960), 168–176. (English and French summaries)

Author's summary: "The relations associating accommodation distance and convergence distance with angle of view are indicated. In addition, the 'perception function' is presented as an equation relating the object distribution in the 'physical' space to that in the 'subjective' space. The significance of this function is discussed."

2281:

**Günther, N. Studien zur Theorie des Raumsehens. III.** Optik **17** (1960), 185–191. (English and French summaries)

Author's summary: "In the two preceding articles formulae relating to the theory of stereoscopic vision were developed. These formulae are now applied to the problem of the elongation of an image arising from oblique observation, which has been considered by E. Lau, and to the physio-psychological effects in the nearer parts of the 'subjective' space."

2282:

**Günther, N. Studien zur Theorie des Raumsehens. IV.** Optik **17** (1960), 278–282. (English and French summaries)

Author's summary: "The 'perception function' is applied to the photogrammetric problem of the degree of exaggeration encountered in the observation of stereoscopic aerial photographs."

2283:

**Kawski, A.; Skwierz, A. Zur Frage des Plattensatzes.** Optik **17** (1960), 203–207. (English and French summaries)

Authors' summary: "The Lunelund recurrence formulae and Stokes' equations expressing the total reflected and transmitted intensity for a pile of plates are modified."

2284:

**Rehmann, G. Über die Ausbreitung von Licht- und Materiewellen in Lorentz-Systemen und ihre geometrische Deutung.** Optik **17** (1960), 98–106. (English and French summaries)

Author's summary: "The geometrical relationships with reference to the propagation of light waves in moving systems of reference are investigated and a geometrical interpretation of the Lorentz contraction is found.—It is shown that a geometrically evident interpretation of the Schrödinger equation can be given and the bearing which this may have on a theory of elementary particles is pointed out."

2285:

**Frank, I. M. The role of group velocity of light emitted in a refractive medium.** Ž. Éksper. Teoret. Fiz. **36** (1959), 823–831 (Russian); translated as Soviet Physics. JETP **9**, 580–586.

The purpose of the paper is to clarify the role of group velocity in the radiation from a source of arbitrary natural frequency moving uniformly in a refractive isotropic medium that is transparent to radiated light. It is shown that equality of the velocity of radiator motion to the group velocity is a necessary condition for the appearance of new radiation components. The analysis predicts the Vavilov-Čerenkov effect, and, for a dispersing medium, several frequencies at which the complex Doppler effect occurs. *D. E. Spencer* (Storrs, Conn.)

2286:

**Dormont, Henri. Optique corpusculaire des pinceaux d'axe courbe.** Ann. Physique (13) **4** (1959), 1341–1387.

This paper is concerned with the motions of electrified particles in an arbitrary static electromagnetic field, and in particular with the properties of sets of trajectories in the neighborhood of a given trajectory (general paraxial ion optics). Focal properties and the formation and defects of images are the principal topics of interest. The subject has been discussed previously, in various degrees of completeness and detail, by other authors, including M. Cotte [Ann. Physique **10** (1938), 333–405], P. A. Sturrock [Philos. Trans. Roy. Soc. London. Ser. A **245** (1952), 155–187; MR **14**, 432], and the reviewer [J. Math. Phys. Mass. Inst. Tech. **20** (1941), 355–369; MR **3**, 214]. The present treatment contains little in the way of detailed mathematical theory; but the author discusses at unusual length the physical assumptions underlying the basic equations, and the immediate physical implications of those equations. A valuable feature of the paper is an extensive review of possible applications of the theory in modern electronic technology.

*L. A. MacColl* (New York)

2287:

**Félici, Noel J. Méthode élémentaire de calcul des lentilles électrostatiques.** J. Phys. Radium **20** (1959), no. 12, supplement, 97A–109A. (English summary)

Author's summary: "The computation of electron lenses has so far been considered as a rather complicated matter, calling for substantial computing facilities. Weak lens formulae were generally discarded by specialists as being unable to give satisfactory results in practical problems.

"The present paper is devoted to a computing method which is, in fact, an extension of the weak lens formulae based on Picht equation. Even in the case of very strong electrostatic lenses, the numerical error does not exceed a few percent. The most tedious part of the computation is the graphical evaluation of an integral, and does not require more than five to ten minutes. This method enables the average scientist or engineer to get a quick and precise insight into the performance of electron lenses.

"In the case of weak lenses, new formulae with series expansions are given, and some interesting properties of those lenses are established."

2288:

**Neufeld, Jacob. Radiation produced by an electron beam passing through a dielectric medium.** Phys. Rev. (2) **116** (1959), 785–787.

Author's summary: "An electron beam passing through a dielectric medium may produce an instability that is associated with the growth of longitudinal waves having a velocity close to the velocity of the beam. For a transparent dielectric medium this instability occurs if the frequency $\omega$ of these waves satisfies the following condition: $\omega_1^2 < \omega^2 < \omega_1^2 + \omega_0^2$, where $\omega_1$ is the frequency of bound oscillators in the dielectric medium and $\omega_0 = (4\pi ne^2/m)^{1/2}$, where $n$ is the electron density. If inhomogeneities are present these longitudinal waves may be converted into transverse waves and radiated into space. Thus, there is a possibility of a luminous effect at 'Bohr frequencies' that differ from the Vavilov-Čerenkov frequencies."

2289:

**Ichikawa, Yoshi H. Stopping power of high temperature plasma. Effects of ionic collective motion.** Progr. Theoret. Phys. **23** (1960), 512–518.

Author's summary: "The problem of energy loss of a charged particle travelling through a fully ionized gas has been studied by taking account of effects of ionic motion. Contribution due to an ionic collective motion turns out to be smaller than that due to an electronic collective motion by order of $m/M$, where $m$ is the mass of electron, $M$ that of ion. The present investigation shows that the ionic collective motion cannot be excited by a charged interloper unless one takes into account effects of thermal motion of electrons."

2290:

**Haines, M. G. The Joule heating of a stable pinched plasma.** Proc. Phys. Soc. **76** (1960), 250–260.

Author's summary: "The hydromagnetic equations are employed to obtain the conditions necessary for a pressure balance in a pinched discharge in ionized deuterium. The

time dependent energy equation is integrated to give the time taken to heat the plasma by Joule heating with bremsstrahlung radiation losses only, and with a pressure balance maintained at all time. This heating time is shown to depend on the radius, line density, and final temperature of the plasma. The current density distribution during the heating process is calculated, showing only a small divergence from uniformity. The condition for no run-away electrons to be present at any time during the heating process is shown to place a restriction only on the minimum line density of the particles. The effect of an unbalance of pressure, causing a change in the outside radius of the plasma during the heating process, is discussed. Finally, the utilization of a transient energy source and its relation to discharge parameters is considered."

2291:

**Fünfer, E.; Lehner, G.; Tuczek, H. Bewegungsvorgänge beim linearen Pincheffekt und Runaway-Elektronen.** Z. Naturforschg. **15a** (1960), 566–574. (English summary)

Authors' summary: "The time dependence of plasma radius and current is calculated on the assumption of homogeneous plasma in the case of the linear stabilised pinch effect. The comparison with the experiment shows a good conformity in regard to the contraction times. Especially it is found by theory and by experiments that the contraction times are independent of the stabilising magnetic field. Measurements of the longitudinal magnetic flux give the temporal and spatial dependence of the induced azimuthal electric field in good agreement with the theory. A deviation at the beginning of the compression is attributed to the paramagnetic effect. A comparison with the theory of Dreiche shows that the measured field strengths are sufficient for the production of runaway-electrons. This gives an explanation for the hard X-rays which are found in the experiments."

2292:

**Anderson, N. Compression waves in a plasma in a static magnetic field.** Proc. Phys. Soc. **75** (1960), 905–912.

Author's summary: "The propagation of small amplitude compression waves through a plasma in a static magnetic field is discussed. In contrast to the usual treatment of this type of problem we shall not treat the ions as a fixed neutralizing background but shall include them in the dynamics of the problem. The problem is discussed in the general case in which the thermal velocity distribution is not negligible. The method of solution is to use the linearized Boltzmann equation in conjunction with Maxwell's equations to obtain a distribution function and dispersion relation. To obtain the dispersion relation an approximation method is used and it is shown that over the greater part of the frequency range the oscillations are of a characteristic wave number. We show that the inclusion of the ions in the dynamics is justified by the fact that at integral multiples of the ion gyrofrequency the ions play the dominant role. The results obtained show that the waves are undamped. The graph of the distribution function for particles with large velocities shows a fine structure which indicates that at these velocities collisions become important, especially for particles moving perpendicular to the field and direction of propagation."

2293:

**Meyer, F.; Schmidt, H. U. Torusartige Plasmakonfigurationen ohne Gesamtstrom durch ihren Querschnitt im Gleichgewicht mit einem Magnetfeld.** Z. Naturforschg. **13a** (1958), 1005–1015. (English summary)

Authors' summary: "Toruslike configurations of a plasma in equilibrium with an exterior magnetic field exist without azimuthal current. The lines of current can be plane curves but their planes cannot intersect in the same axis. The plasma cross-section must have maxima and minima. Azimuthal and meridional cross-sections are calculated.—From the equilibrium conditions the construction rule for a deformable (paper) model network is derived. This network is a true analogue to the totality of equilibrium surfaces of a plasma without an interior field. It is applied to a torus configuration."

*C. H. Papas* (Pasadena, Calif.)

2294:

**Argyres, P. N.; Roth, L. M. Theory of electrical conduction in high magnetic fields.** Phys. and Chem. Solids **12** (1959/60), 89–96.

Authors' summary: "It is pointed out that previous quantum-mechanical theories of transport in high magnetic fields are deficient in that they neglect the effect of the electric field on scattering. It is demonstrated here that in the case of large Hall angles, i.e., $\omega_0\tau \gg 1$, the transverse current can be obtained by a direct expansion in powers of the scattering potential. Both elastic and inelastic collisions are considered. It is found that the transverse current can be described in terms of the drift of the centers of cyclotron orbits of the electrons in the magnetic field. This justifies the original semi-classical method of calculation of Titeica. It is pointed out, however, that such a procedure is correct only for a non-oscillating electric field. No applications are made."

2295:

**Vandakurov, Yu. V.; Perel', V. I. The motion of positive ions in a natural gas under the effect of electric and magnetic fields.** Ž. Tehn. Fiz. **29** (1959), 958–961 (Russian); translated as Soviet Physics. Tech. Phys. **4** (1960), 871–874.

Authors' summary: "The velocity distribution function is obtained for ions which move in a natural gas in electric and magnetic fields for the case in which the mean energy of the ions is much larger than the mean energy of the neutral atoms. It is assumed that the main interaction mechanism between the ions and the atoms is resonance charge exchange."

2296:

**Drummond, W. E.; Rosenbluth, M. N. Cyclotron radiation from a hot plasma.** Phys. Fluids **3** (1960), 45–51.

2297:

**Gambirasio, Giorgio. On the electrical behavior of an ideal plasma.** Phys. Fluids **3** (1960), 299–302.

Author's summary: "The solution of the equation for the current in an ideal plasma, when the electric field, in a direction perpendicular to a constant magnetic field, is

abruptly increased from zero to a constant value, is obtained using Laplace transforms. An exact solution and approximate solutions for some simple cases are obtained and discussed. An expression and the corresponding RLC network are found for the specific impedance of the plasma."

2298:

**Reagan, Daryl. Transverse compression waves in a stabilized discharge.** Phys. Fluids **3** (1960), 33–39.

Author's summary: "An electric discharge which compressed by its own magnetic field, and 'stabilized' by means of an axial magnetic field, can have transverse wave motions which cause its periodic compression and expansion. This kind of motion can cause the heating of the ions in the discharge. The simplest of these wave modes are described and an estimate is given of the power available to the waves as a result of the interaction of the electrons in the discharge with an axial electric field. This interaction can cause the attenuation or spontaneous growth of the waves, depending upon the circumstances. It is likely that in high current gas discharge experiments there are examples of growing and decaying waves of this type."

2299:

**Schirmer, H.; Friedrich, J. Die Wärmeleitfähigkeit eines Plasmas.** Z. Physik **153** (1959), 563–570.

Authors' summary: "Es wird die Lösung der Boltzmann-Gleichung eines Plasmas (beliebigen Ionisierungsgrades) nicht gleichförmiger Temperatur angegeben. Sie führt zur Darstellung der Stromdichte und des Wärmestroms der Elektronen; damit sind Wärmeleitfähigkeit, Beweglichkeit, Diffusion und Thermodiffusion der Elektronen erfaßt. Alle diese Transportkoeffizienten lassen sich durch die Größen $\bar{l}_v$ und $\bar{l}_\lambda$ (als 'mittlere freie Weglängen') sowie durch eine Kopplungsgröße $A$ beschreiben, die—bei vollständiger Übereinstimmung der Ausdrücke der Transportkoeffizienten für Lorentz-Gas und Plasma—für ein Lorentz-Gas durch Integral, für ein Plasma dagegen durch Determinanten darstellbar sind."

*C. H. Papas* (Pasadena, Calif.)

2300:

**Larkin, A. I. Passage of particles through a plasma.** Ž. Èksper. Teoret. Fiz. **37** (1959), 264–272 (Russian); translated as Soviet Physics. JETP **10** (1960), 186–191.

Author's summary: "The Green's function method and the diagram technique are used to calculate the energy loss per unit time by a particle passing through a plasma. Numerical values of the factors in the argument of the logarithm have been obtained for limiting cases."

2301:

**Hruška, A. The collisions of two plasma streams.** Czechoslovak J. Phys. **10** (1960), 33–39. (Russian summary)

2302:

**Schumann, Winfried Otto. Über die Entstehung einer "Backward Wave" in einem nichtmagnetisierten, von Luft begrenzten Plasmazylinder.** Z. Angew. Phys. **12** (1960), 145–148.

2303:

**Kirstein, P. T. A paraxial formulation of the equations for space-charge flow in a magnetic field.** J. Electronics Control (1) **8** (1960), 207–225.

Author's summary: "The equations for irrotational, axially symmetric, laminar, space-charge flow are set up in a paraxial manner. To this approximation, the flow is specified by the magnetic field configuration, one trajectory, the potential along this trajectory, and the variation of beam thickness. Any three of these quantities may, within certain limits, be specified arbitrarily, and the fourth then computed by the formulae given. The method is directly applicable to flows in which the cathode is conical, and either the flow lines do not cut magnetic flux lines, or the magnetic field is tangential at the cathode. Numerical results are given for a beam from a cylindrical cathode, and extensions of the methods are discussed."

2304:

**Poincelot, Paul. Conséquences tensorielles des équations de Maxwell.** Ann. Télécommun. **14** (1959), 106–110.

2305:

**Lange, F. H. Entwicklungstendenzen der modernen Operatorenrechnung.** Hochfrequenztech. Electroak. **69** (1960), 67–75.

2306:

**Vigoureux, P. Development of the formulae of electromagnetism in the M.K.S. system.** Proc. Inst. Elec. Engrs. B **107** (1960), 331–340.

2307:

**Müller, F. Der Gültigkeitsbereich der Gleichung** rot rot $\mathfrak{v} = \text{grad div } \mathfrak{v} - \Delta\mathfrak{v}$. Hochfrequenztech. Electroak. **69** (1960), 62–67.

2308:

**Debever, Robert. Tenseur de super-énergie et composantes irréductibles du tenseur de Riemann.** C. R. Acad. Sci. Paris **250** (1960), 64–66.

A super-energy $V_{\alpha\beta\lambda\mu}$ is expressed in terms of the Riemann tensor $R_{\alpha\beta\lambda\mu}$ if space-time has a hyperbolic metric. In the case of an electromagnetic field, $R_{\alpha\mu} = R_{\alpha\beta}{}^{\beta}{}_{\mu}$ is identified, within a constant factor, with the momentum-energy tensor $T_{\alpha\mu}$. Applications are given to the singular and non-singular electromagnetic fields. The Riemann tensor is entirely characterized by conditions expressed in terms of fields of isotropic vectors. *D. E. Spencer* (Storrs, Conn.)

2309:

**Richter, E. Zur Theorie kraftfreier Magnetfelder.** Z. Physik **159** (1960), 194–211. (English summary)

Author's summary: "Force-free magnetic fields are defined by the equation rot $\mathfrak{H} = \alpha\mathfrak{H}$. Making use of a moving Frenet coordinate system ($\mathfrak{t}$ = tangential, $\mathfrak{n}$ = normal, $\mathfrak{b}$ = binormal unit vector) we find the following general features of these fields: 1. grad $|\mathfrak{H}|$ is always parallel to the osculating plane of the $\mathfrak{H}$-lines. 2. If the lines

of force are rectilinear within a finite region of space, the component of grad $|\mathfrak{H}|$ along $\mathfrak{H}$ must be zero for a force-free field with rot $\mathfrak{H} \neq 0$. 3. The factor of proportionality $\alpha = \alpha(\mathfrak{r})$ and $|\mathfrak{H}|$ can be calculated by means of two equations involving only the direction of $\mathfrak{H}$. For several models of force-free fields the effect of symmetry assumptions on $\alpha = \alpha(\mathfrak{r})$ is discussed using special coordinate systems. In the appendix it is pointed out that a particle drift arises in magnetic fields with $\alpha \neq 0$."

2310:

**Chu, E. L. The Lagrangian and the energy-momentum tensors in the perturbation theory of classical electrodynamics.** Ann. Physics **9** (1960), 76–92.

The Lagrangian density of a charge-field system is expanded in a power series of the perturbation parameter $\varepsilon$ by means of an infinitesimal transformation from the space-time point, $\tilde{r}$, of the perturbed electron to $\tilde{r}_0$ of its unperturbed state. The higher order Lagrangian functions, which are the coefficients of this expansion, are worked out to order $L^{(2)}$ and the question considered whether these functions characterize closed physical systems.

In the case of the first-order function it is shown that the stationary property of the corresponding action integral leads to a symmetric energy-momentum tensor. For second and higher order functions a somewhat different procedure is used. The indicated transformation and expansion are applied to the field and particle members of the equation satisfied by the energy-momentum tensor. Each term of the perturbation expansion for this tensor is then shown to be symmetric. It is thereby concluded that the Lagrangian of any order in the unperturbed coordinates characterizes a closed physical system.

*R. D. Kodis* (Providence, R.I.)

2311:

**Itzkan, I. Solutions of the equations of space charge flow for radial flow between concentric spherical electrodes.** J. Appl. Phys. **31** (1960), 652–655.

Author's summary: "The solutions of the equations of space charge flow of electron beams for radial flow between concentric spherical electrodes are formulated in such a manner as to permit the introduction of arbitrary initial conditions. This allows one to solve a variety of physical problems with the aid of a table of Airy functions. A particular case, which exhibits a potential minimum between electrodes, is presented as an illustrative example."

*J. C. P. Miller* (Cambridge, England)

2312:

**Durand, Philippe. Développement en série du potentiel au voisinage d'un coin diélectrique rectangulaire et recherche d'une équation aux différences finies.** C. R. Acad. Sci. Paris **250** (1960), 1983–1985.

2313:

**Power, G.; Jackson, H. L. W. On problems associated with two-dimensional fields in homogeneous anisotropic media.** Acta Phys. Austriaca **13**, 129–139 (1960).

The authors show how the familiar complex variable methods for solving two-dimensional electrostatic boundary value problems may be extended so as to apply to piecewise homogeneous anisotropic dielectric media. They also show how the resultant electrostatic force and moment on any region of a dielectric, as calculated from the Minkowski form of the electrostatic stress tensor in a polarized dielectric medium, can be expressed simply and elegantly in terms of complex potentials. Specific boundary value problems are solved to illustrate the methods.

*R. A. Toupin* (Washington, D.C.)

2314:

**Cox, J. A. M. Čerenkov radiation.** Nederl. Tijdschr. Natuurk. **26** (1960), 204–215. (Dutch)

2315:

**Fulton, Thomas; Rohrlich, Fritz. Classical radiation from a uniformly accelerated charge.** Ann. Physics **9** (1960), 499–517.

The authors undertake a critical study of various questions relating to radiation from a uniformly accelerated electrified particle. After having shown that the particle does radiate at a constant rate, the authors state that there is no well established and generally accepted differential equation of motion for the radiating particle. The equation which they appear to regard as being the most plausible contains no term representing a reaction of the radiation on the particle. This has the unsatisfactory consequence that all of the work done by the applied force appears as kinetic energy of the particle, the energy carried away by radiation being unaccounted for. No clear means of escape from this dilemma is suggested; and the final conclusion is that the fundamental questions remain difficult and open.

*L. A. MacColl* (New York)

2316:

**Trenev, N. G. Diffraction of surface electromagnetic waves on an impedance step.** Radiotehn. i Elektron. **3** (1958), 27–37. (Russian)

2317:

**Loges, F. Eine Näherungslösung für die Beugung einer ebenen monochromatischen Welle am Spalt.** Optik **17** (1960), 75–83. (English and French summaries)

Author's summary: "The two-dimensional problem of the diffraction of a plane-polarised, plane wave incident in any manner on a slit is approximately solved. The approximation, which is given in the form of integrals of the wave equation in the coordinates of the elliptic cylinder, does not conform to the boundary condition. The resulting error is made negligible by means of suitable integrals of the emergent waves at the edges of the slit. Applications are indicated for normal incidence."

2318:

**Westpfahl, Konradin. Zur Theorie einer Klasse von Beugungsproblemen mittels singulärer Integralgleichungen. I. Teil A. "Klassische" Beugungsprobleme.** Ann. Physik (7) **4** (1959), 283–351.

Author's summary: "Für eine Klasse zweidimensionaler Beugungsprobleme (Halbebene, Spalt, Gitter usw.) wird die Randwertaufgabe auf singuläre Integralgleichungen vom Cauchyschen Typ zurückgeführt (Randbedingung $v = 0$ auf dem Schirm). Die Anzahl der Gleichungen stimmt

mit der Anzahl der Kanten des Schirms überein. Für das Einkantenproblem läßt sich die Integralgleichung mittels der funktionentheoretischen Methode von Muskhelishvili geschlossen auflösen (die Methode enthält das Wiener-Hopf-Verfahren als Spezialfall). Das Zweikantenproblem wird asymptotisch (für einen gegenüber der Wellenlänge großen Kantenabstand) gelöst und numerisch mit der Reihenentwicklung nach Mathieuschen Funktionen verglichen.

"Die mathematischen Methoden zur Behandlung 'technischer' Beugungsprobleme (offene und geschlitzte Wellenleiter) werden bereitgestellt. Eine Zusammenstellung der mathematischen Ergebnisse findet sich im Anhang."

*E. T. Copson* (St. Andrews)

2319:

**Pimenov, Yu. V. The plane problem of the diffraction of electromagnetic waves by two ideally conducting strips of finite width, located one below the other.** Ž. Tehn. Fiz. **29** (1959), 711–715 (Russian); translated as Soviet Physics. Tech. Phys. **4**, 638–642.

An approximate solution of the problem described in the title is obtained by formulating it as two independent integral equations

$$\pi i \int_{-1}^{1} u_{\pm}(\xi)\{H_0^{(2)}(ka|\zeta-\xi|) \pm H_0^{(2)}(ka\sqrt{(h^2+(\zeta-\xi)^2)}\}d\xi = 2F_{\pm}(\zeta),$$

where $-1<\zeta<1$, and then using a method of successive approximations. But as the author does not attempt to prove the convergence of his method, the analysis can only be regarded as very formal.

*E. T. Copson* (St. Andrews)

2320:

**D'yakonov, B. P. Diffraction of electromagnetic waves on a sphere in a half-space.** Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 1579–1590. (Russian)

2321:

**Kovalenko, E. S. The gyrotropic elliptical waveguide.** Dokl. Akad. Nauk SSSR **128** (1959), 276–279 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1003–1006.

2322:

**Adachi, Saburo. Impedance characteristics of a uniform current loop having a spherical core.** J. Res. Nat. Bur. Standards Sect. D **64D** (1960), 295–299.

2323:

**Kron, Gabriel. ★Tensors for circuits.** 2nd ed. With an introduction by Banesh Hoffmann. Dover Publications, Inc., New York, 1959. xxvii+250 pp. $1.85.

The main text of this book is a second printing of the author's earlier book, *A short course in tensor analysis for electrical engineers* [Wiley, New York, 1942; MR **4**, 29]. The second printing is a corrected version of the earlier work and has an extensive compilation of the published works of the author.

The reviewer is glad to see that the author's book, dealing with algebraic topological methods, is once again available to teachers, students and those practising in the engineering sciences. *A. A. Mullin* (Urbana, Ill.)

2324:

**Samoĭlenko, Yu. I. On the theory of autooscillation synchronization by small external forces in a system with n degrees of freedom.** Radiotehn. i Elektron. **3** (1958), 1361–1372. (Russian)

## CLASSICAL THERMODYNAMICS, HEAT TRANSFER

See also 2226, 2227, 2278, 2457.

2325:

**Erdélyi, I. Chapters on the mechanical theory of heat.** Acta Tech. Acad. Sci. Hungar. **27** (1959), 127–145. (German, French and Russian summaries)

2326:

**Andriankin, È. I. Heat wave, radiating energy from front.** Ž. Tehn. Fiz. **29** (1959), 1368–1372 (Russian); translated as Soviet Physics. Tech. Phys. **4** (1960), 1258–1262.

Author's summary: "In this article, we investigate the propagation of a nonprogressive heat wave, radiating energy from the front. We examine the case for which the path of the radiation in the cold gas is large for all frequencies below the critical frequency $\omega_*$, and is small for high frequencies. The path of the quanta in the heated region is assumed to be much smaller than the radius of the wave front, since the radiant energy transfer takes place by means of heat conduction."

2327:

**Agar, J. N. The rate of attainment of Soret equilibrium.** Trans. Faraday Soc. **56** (1960), 776–787.

Author's summary: "Equations giving the rate of change of concentration during thermal diffusion in 2-component liquid systems are developed, starting from the usual form of Fick's law and avoiding the ambiguities in the specification of frames of reference and diffusion coefficients that occur in earlier treatments. Volume changes during non-isothermal diffusion (with steady temperatures) are also considered and shown to be negligible under ordinary conditions.

"A simplified diffusion equation, valid for small temperature intervals, is derived and solutions are given in the 'trigonometric' and in the less familiar 'error function' forms; the latter are convenient for short times.

"Corrections arising from changes in the temperature gradient during the 'warming-up' period at the beginning of a thermal diffusion experiment are evaluated; they are small but not always negligible."

2328:

**Thomas, P. H. Some approximations in the theory of self-heating and thermal explosion.** Trans. Faraday Soc. **56** (1960), 833–839.

Author's summary: "In transient self-heating problems the spatial distribution of temperature in symmetrically heated bodies is usually assumed to be uniform and an effective surface transfer coefficient is defined so that the analysis can be simplified. This paper shows how this effective heat transfer coefficient can be simply related to the real surface cooling."

2329:
**Planck, Max. ★The theory of heat radiation.** Authorised translation by Morton Masius. Dover Publications, Inc., New York, 1959. xiv+224 pp. Paperbound: $1.50.

This is a translation of the second (1913) edition of Max Planck's "Waermestrahlung".

2330:
**Gurtin, M. E. On the use of normal coordinates for the solution of lumped parameter transient heat-transfer problems.** J. Aero/Space Sci. **27** (1960), 357–360.

2331:
**Citron, Stephen J. Heat conduction in a melting slab.** J. Aero/Space Sci. **27** (1960), 219–228.

An infinite slab is insulated on one side and subjected to heat input on the other side. At a certain fixed temperature melting sets in on this side and continues into the slab. Initial temperature in the slab and time-dependent heat input are arbitrary. Instantaneous removal of molten liquid and temperature-independent thermal conductivity are assumed.

The problem is of the one-dimensional Stefan type (heat conduction in a body with moving boundaries) and is reduced to a nonlinear differential equation. An iteration process for obtaining an approximate solution is suggested. Nothing is said about convergence but the numerical example presented seems to corroborate convergence of the procedure. *H. Parkus* (Vienna)

2332:
**Mason, E. A.; Saxena, S. C. Approximate formula for the thermal conductivity of gas mixtures.** Phys. Fluids **1** (1958), 361–369.

Authors' summary: "An approximate formula for the thermal conductivity of multicomponent gas mixtures is derived from rigorous kinetic theory by well-defined approximations. Numerical calculations with the formula are relatively simple, and the only data needed are the molecular weights, thermal conductivities, and either viscosities or heat capacities of the pure components at the same temperature as the mixture. The form of the formula is quite similar to the earlier empirical Lindsay-Bromley equation. The formula is tested by comparison with experimental results on a number of binary and ternary mixtures involving both monatomic and polyatomic nonpolar gases. Agreement is satisfactory, and is nearly as good as obtained with the full rigorous theory."

2333:
**Sidorov, È. A. On the interaction of convection and radiation in an absorbing medium.** Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 5, 134–136. (Russian)

2334:
**Hirschfelder, Joseph O. Diffusion coefficients in flames and detonations with constant enthalpy.** Phys. Fluids **3** (1960), 109–112.

## QUANTUM MECHANICS

See also A1818, 2052, 2068, 2081, 2188.

2335:
**Gel'man, A. P. On the relativistic operators for momentum and angular momentum.** Ž. Èksper. Teoret. Fiz. **37** (1959), 477–481 (Russian); translated as Soviet Physics. JETP **10** (1960), 339–341.

The author observes that the form of the relativistic momentum operator $-i\hbar\tilde{D}_k$, as applied to spinors, which has sometimes been used in calculations referred to (orthogonal) curvilinear coordinates, is not correct in the sense that the index $k$ does not possess covariant character. The correct form $-i\hbar D_k$ of the canonical momentum operator is derived here, and unlike $\tilde{D}_k$, $D_k$ is not diagonal. The reason why the use of $\tilde{D}_k$ leads to correct results in the case of Dirac's equation is discussed. The angular momentum operators are also considered.

*H. A. Buchdahl* (Hobart)

2336:
**Sen, P. The renormalization of Dirac-Maxwell equations.** Nuovo Cimento (10) **13** (1959), 1122–1132. (Italian summary)

2337:
**Wegener, Horst. Schrödinger equation in multiply connected spaces and phase optics.** Z. Physik **159** (1960), 243–247.

Author's summary: "The magnetic vector potential **A** in a field free space $R_0$ cannot be removed by gauge transformations in general, if $R_0$ is multiply connected. Aharonov and Bohm [Phys. Rev. (2) **115** (1959), 485–491; MR **22** #1336] have noticed recently that **A** therefore should have more physical meaning than only to give the magnetic field by differentiation. They could show that **A** in $R_0$ may influence the phase of Schrödinger's $\psi$-function in an observable manner. We want to point out here that this influence can be expressed in a simple, general form: 'A closed magnetic field line operates upon $\psi$ like a $e\Theta/\hbar$-phase-shifter placed on any area bounded by the field line.' Surface-like phase shifters are familiar in phase optics. There exists a narrow relationship between electron scattering at magnetic fields and some special problems of phase optics. An electron phase contrast microscope is discussed."

2338:
**De Alfaro, V. On the inversion problem for a Klein-Gordon wave equation.** Nuovo Cimento (10) **10** (1958), 675–681. (Italian summary)

Author's summary: "We treat in this paper the problem of obtaining the static potential which corresponds to given resonance parameters in the case of a Klein-Gordon wave equation. Our results are an extension of those already found in the non-relativistic case. We restrict the class of our potentials to those vanishing outside a finite range $a$. The knowledge of the resonance parameters allows one to obtain an approximate solution of the so-called Gel'fand-Levitan equation which provides the desired potential." *M. Cini* (Rome)

2339:

**Schlögl, Friedrich. Die Mannigfaltigkeit informationsfremder Zustände in der Quantentheorie.** Z. Physik **159** (1960), 411–427.

Author's summary: "Es wird untersucht, wie weit die Annahmen, die in der Quantentheorie auf die Darstellbarkeit der Observablen durch lineare Operatoren führen, sich zwangsläufig aus der Existenz nicht gleichzeitig meßbarer Größen ergeben. Dabei scheint eine fundamentale Annahme insofern willkürlich zu sein, als sie nur ein Einfachheitspostulat darstellt. Nach diesem lassen quantenmechanische Systeme, deren Zustandsraum mehr als zwei Dimensionen besitzt, nur jeweils zwei zueinander informationsfremde maximale Messungen zu. Diese Tatsache führt auf die Auszeichnung komplementärer Meßgrößen, die sich auch bei den hier allein betrachteten endlich-dimensionalen Zustandsräumen definieren lassen. Alle Observablen eines Systems lassen sich als Funktionen dieser komplementären Größen darstellen."

2340:

**Lurié, D. On the Pauli group.** Physica **25** (1959), 1139–1141.

Author's summary: "An eight-dimensional generalization of Wouthuysen's six-dimensional scheme for fermions is seen to result if use is made of the three-parameter Pauli group." *A. C. Hurley* (Melbourne)

2341:

**Davies, H. On the convergence of the Born approximation.** Nuclear Phys. **14** (1959/60), 465–471.

Author's summary: "It is shown that the Born series for the non-relativistic scattering amplitude $f(k)$ by a spherically symmetric static potential $V(r)$, which is bounded and has finite range, is convergent for all $k$ if the potential $-|V(r)|$ cannot support a bound state."
*D. F. Mayers* (Oxford)

2342:

**Menotti, P. Wigner's inequalities for relativistic scattering.** Nuclear Phys. **14** (1959), 232–237.

Author's summary: "Wigner's inequalities for the momentum derivatives of the scattering phases are extended to the relativistic scattering of particles of spin zero and one half by a central potential of finite range."
*K. Johnson* (Cambridge, Mass.)

2343:

**Michel, L. Covariant description of polarization.** Nuovo Cimento (10) **14** (1959), supplemento, 95–104.

The author starts from the realization of the irreducible representation of mass $m$ and spin $s$ given by Wigner, in which the state is represented by a $(2s+1)$-component vector function of $p$, the four momentum. He then introduces the $(2s+1)\times(2s+1)$ density matrix of such a state and describes its transformation law under Lorentz transformations and rotations. He shows for mass zero how the density matrices can be parametrized by the points of the Poincaré sphere. The infinitesimal operators of the inhomogeneous Lorentz group are then introduced and the density matrix is expressed in terms of them for arbitrary mass and spin. The resulting formulae are given in detail for the Dirac theory of a spin 1/2 particle, both $m\neq 0$ and $m=0$, and for the Bergmann-Wigner theory of particles of arbitrary spin. *A. S. Wightman* (Princeton, N.J.)

2344:

**Purkayastha, Sabita. Derivation of the wave functions for $j=l+1/2$, $j=l-1/2$ states and for '0' spin using Kar's linear Hamiltonian.** Indian J. Theoret. Phys. **6** (1958), 97–106.

2345:

**Tzou, K. H. Inversion de masse et solutions des équations de Dirac.** J. Phys. Radium **20** (1959), 933–936. (English summary)

Author's summary: "In Dirac's theory, solutions generated by the mass reversal operation $M$ and by the operators $MG$ are identified, $G$ being the symmetry group $\{1, P, T, C$, and combinations$\}$."

2346:

**Datzeff, A. B. Sur l'interprétation de la mécanique quantique. I. Une nouvelle hypothèse. Conséquences qualitatives.** J. Phys. Radium **20** (1959), 949–955. (English summary)

Author's summary: "A causal interpretation of quantum mechanics is sought, deriving from the hypothesis that the field $U$ has a material support (subvac) with a discrete structure of AS corpuscules. The latter can group into stable formations $\Phi_k$ ($k=1, 2, \cdots$). Interacting with $\Phi_k$ a microcorpuscule $\mu$ (electron, etc.) is in a static condition $A_k$, with energy $E_k$. Because of the fluctuations of $\Phi_k$ and AS the movement of the corpuscule $\mu$ should not be described by means of classical mechanics but statistically. It is shown that in a number of cases one obtains, for the probability of the presence of $\mu$, qualitatively the same results as given by quantum mechanics.

"To find an exact quantitative description of the statistical distribution of the positions of $\mu$ one should determine, in all cases, the probability of the presence $w(x, y, z)$ of $\mu$, which will be given in a paper to follow."

2347:

**de Broglie, Louis. L'interprétation de la mécanique ondulatoire.** J. Phys. Radium **20** (1959), 963–979. (English summary)

Author's summary: "Restatement of the interpretation of wave mechanics by the double-solution theory, which the author, beginning again his attempts of 1924–1927, tried to develop with the collaboration of several young scientists.

"Some results recently obtained are specially made obvious, in connection with new points of view, which commanded the author's attention since his statements of the years 1954–1955."

2348:

**Sokolov, A. A.; Ternov, I. M.; Loskutov, Ju. M. On the problem of transformation properties of the spin pseudovector.** Ann. Physik (7) **5** (1960), 241–248.

2349:

**Aržanyh, I. S. On an algorithm for quantum mechanics.** Dokl. Akad. Nauk SSSR **126** (1959), 45–48 (Russian); translated as Soviet Physics. Dokl. **4**, 558–561.

This work proposes and discusses an algorithm for obtaining a quantum-mechanical hamiltonian from a given classical one. It is novel both in its formal character and in its applicability to relativistic as well as non-relativistic case. On the other hand the physical motivation and meaning for the algorithm are largely untreated, and the mathematical treatment is entirely formal.

*I. E. Segal* (Cambridge, Mass.)

2350:

**Hamilton, J. Dispersion relations for $p$-$n$ scattering.** Phys. Rev. (2) **114** (1959), 1170–1178.

Author's summary: "The application of dispersion relations to low-energy $p$-$n$ scattering is examined. It is shown that Khuri's dispersion relation can be extended to include tensor forces, but serious difficulties appear on attempting to include exchange forces. The application of the relativistic field theory dispersion relations to low-energy scattering is made by using the effective range formula. The spurious poles of the $S$-matrix are related to the two- and three-pion terms in the unphysical region contribution for forward scattering."

2351:

**Ford, Kenneth W.; Wheeler, John A. Semiclassical description of scattering.** Ann. Physics **7** (1959), 259–286.

Authors' summary: "The quantum-mechanical scattering amplitude can be simply related to the classical deflection function when the conditions for a semiclassical analysis of the quantum-mechanical scattering are met. Various interesting characteristic features of the scattering are related to special features of the classical deflection function. The characteristic types of scattering discussed are: interference, when the deflection function possesses more than one branch at a given angle; rainbow scattering, when the deflection function has a relative maximum or minimum; glory scattering, when the deflection function passes smoothly through $0°$ or through an integral multiple of $\pm\pi$; and orbiting, when the deflection function possesses a singularity. The consideration of the characteristic features of semiclassical scattering makes possible the analysis of an observed differential cross section to yield the classical deflection function, which in turn may be used to construct the potential."

2352:

**Ford, Kenneth W.; Wheeler, John A. Application of semiclassical scattering analysis.** Ann. Physics **7** (1959), 287–322.

Authors' summary: "Several different examples of scattering processes are analyzed, and for each, semiclassical approximations are discussed. The scattering of magnetic monopoles by charged particles is an illustrative example demonstrating rainbow and glory effects. In this example, exact classical and quantum results are also obtained and comparison made with semiclassical results. For the scattering of alpha particles by nuclei, the influence of absorption and of the phenomenon of orbiting on the cross section are studied semiclassically, and comparison with experiment is made. For the scattering of atoms by atoms, rainbow, glory, and orbiting effects may all exist, and one example is worked out semiclassically in detail. The scattering of electrons or muons by nuclei is discussed briefly and the semiclassical analysis shown not to be valid at any energy."

2353:

**Zav'yalov, O. I. Dispersion relations and perturbation theory.** Dokl. Akad. Nauk SSSR **128** (1959), 273–275 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1000–1002.

2354:

**Percival, I. C. A variational principle for scattering phases.** Proc. Phys. Soc. **76** (1960), 206–216.

Author's summary: "A first-order variational principle for a stationary scattering phase is obtained by keeping constant the energy normalization for permitted variations of the wave function. The principle is used to derive (i) a variational correction to the phase obtained from given approximate wave functions, (ii) a first-order perturbation theory, (iii) a first Born approximation and (iv) a variational method for obtaining approximate wave functions analogous to that of Kohn. (i), (ii) and (iii) are more generally applicable than the corresponding expressions for $\tan\delta_l$ and $\sin\delta_l$, where $\delta_l$ is the phase, and simple models suggest that they are more accurate than that for $\tan\delta_l$, but an example indicates that (iv) may be worse than Kohn's variational method. A possible explanation is given.

"The theory is generalized to many channels. Many complications arise which are absent or trivial for one channel, and the generalization is not complete, since we are unable to define a stationary phase matrix, and the range of application is more limited."

2355:

**Young, James E. Three-body breakup; deuteron dissociation cross sections.** Phys. Rev. (2) **116** (1959), 1201–1211.

2356:

**Sucher, J. An asymptotic causality requirement for systems with constrained inputs.** Nuclear Phys. **14** (1959), 263–269.

The causality condition relating input and output of linear systems usually is formulated as an integral transform the kernel of which characterizes the system. The condition postulates "no output before input". This condition does not necessarily imply that the kernel $K$ is a causal function of the time variable $t$ because the set of admissible causal input functions might be incomplete or even empty. Then the causality requirement would be inconsequential. An attempt is made to formulate this requirement in an asymptotic way which overcomes this difficulty.

*B. Gross* (Rio de Janeiro)

2357:

**Federbush, P. Haag's theorem in a finite volume.** Nuovo Cimento (10) **15** (1960), 932–933. (Italian summary)

Author's summary: "The question of extending Haag's theorem to a finite volume is explored. A positive result is suggested."

2358:

**Mandl, F. ★Introduction to quantum field theory.** Interscience Publishers, New York-London, 1959. vii+202 pp. $6.00.

This is an elementary text which discusses free fields, the $S$-matrix expansion in terms of Feynman graphs, simple scattering calculations, and the elements of renormalization theory. The style is pleasant and lucid. The book provides an excellent introduction to quantum field theory and will be particularly useful to those who want to learn some of the simpler techniques without getting involved in more modern and sophisticated developments.

*J. C. Polkinghorne* (Cambridge, England)

2359:

**Rollnik, H.; Stech, B.; Nunnemann, E. Quantenelektrodynamik und Asymptotenbedingung.** Z. Physik **159** (1960), 482–494. (English summary)

Authors' summary: "The asymptotic conditions for the field operators are used in order to employ only renormalized quantities in quantum electrodynamics and to avoid the usual procedure of adiabatic switching off of the interaction. The treatment is based mainly on invariance properties and therefore gives a somewhat simpler formulation of the renormalization procedure."

2360:

**Wilhelmsson, H.; Kerman, A. On the integration of a certain matrix element $\langle 0|f^{(2)}(x)|q, k\rangle$ in the perturbation approach of quantum electrodynamics.** Ark. Fys. **17**, 149–156 (1960).

The $e\cdot\alpha$ term of the matrix element $\langle 0|f(x)|q, k\rangle$ where $|q, k\rangle$ is the incoming scattering state of an electron and photon and where $f(x)=(\gamma^\mu\partial_\mu+m)\psi(x)$, is calculated.

*K. Johnson* (Cambridge, Mass.)

2361:

**Schwinger, Julian. Euclidean quantum electrodynamics.** Phys. Rev. (2) **115** (1959), 721–731.

Author's summary: "Quantum electrodynamics is transcribed into a Euclidean metric. A review is presented of the quantum action-principle approach to quantization, with its automatic emphasis on the dynamical variables associated with the physical degrees of freedom. Green's functions of the radiation gauge are defined, and then characterized by differential equations and boundary conditions. These Green's functions are of direct physical significance but involve a distinguished time-like direction. A gauge transformation is then performed to eliminate this dependence, introducing thereby the Green's functions of the Lorentz gauge, which lack immediate physical interpretation. The latter functions are now primarily defined by differential equations and boundary conditions, and form the basis for the analytic extension which is the change from space-time to Euclidean metric. Some properties of anticommuting matrices are discussed in relation to this metric transformation. Real Euclidean Green's functions are defined by correspondence with the Lorentz gauge functions and the appropriate differential equations obtained. Invariance properties of the Euclidean functions are discussed. The individual Euclidean Green's functions are given an operator construction and then combined into a generating Green's functional which is interpreted as the wave function, in a canonical field representation, of a state characterized by the Euclidean action operator. Differential operator realizations and some other benefits of a canonical variable description are exhibited."

*S. Deser* (Waltham, Mass.)

2362:

**Schwinger, Julian. Field theory of unstable particles.** Ann. Physics **9** (1960), 169–193.

The author believes that the description of unstable particles is already fully contained in the general theory of Green's functions, and sets out to develop this theory in such a way as to emphasize the relevant properties, using the spinless boson field as an example. The structure of the spectral representations of the Green's function $\mathfrak{G}(\kappa^2)$ in the case where there are $n$ stable particles associated with the field is examined in detail. In certain circumstances a rapid variation in the phase of $\mathfrak{G}$ is indistinguishable from a discontinuity corresponding to a pole or zero. An unstable particle is described by a Green's function with such a region of rapid phase variation. It is shown that so long as this region dominates over the thresholds the decay law of such a particle will be exponential. By consideration of an idealized experiment to determine the presence of an unstable particle—a mass filter—it is shown that the limit of the exponential decay law is also the limit of applicability of the concept of an unstable particle.

*T. W. B. Kibble* (London)

2363:

**Rayski, Jerzy. A six-dimensional Riemannian manifold, its applications to meso-electrodynamics, and a systematization of strongly interacting particles.** Acta Phys. Polon. **18** (1959), 371–385.

The "non-geometrical" conservation laws (e.g., of charge and baryon number) are given a geometrical interpretation (as intrinsic spins) by adding two dimensions to four-space. Rotations in the new subspace are related to gauge transformations and so to the electromagnetic field. Similarly, isotopic spin and the pseudo-scalar nature of the pion are related to the Dirac algebra in these six dimensions. The usual charge-strangeness relation is rewritten in terms of new quantum numbers (e.g., the new "baryon" number is non-zero for $K$-mesons), which again are related to intrinsic spins in other subspaces of the six space. Finally, the elementary particles are systematized according to these concepts, and it is suggested that their masses arise as eigenvalues from the new quantum numbers. A suggestion of how the observed mass ratios arise is given in terms of a model in which a particle is a "rigid rotor" in a two-dimensional time-like subspace.

*S. Deser* (Waltham, Mass.)

2364:

**Sharp, R. T. Self and external meson fields.** Canad. J. Phys. **37** (1959), 515–520.

Author's summary: "Instead of a single (isotopic vector) field to describe pions, it is sometimes advantageous to introduce several such fields with identical properties except for their coupling to the sources of the field (the baryons). In this way one can formally distinguish between self pions (roughly, those which can be emitted and absorbed by the same baryon) and external pions (roughly, those which are only exchanged between baryons or emitted into or absorbed from free states). $K$-mesons can be treated similarly. The device, which is of general applicability, simplifies many derivations and calculations in quantum field theory. As an illustration of the method it is used to derive the Low equations for scattering of pions and $K$-mesons by nucleons and for associated production. A suggestion is made for treating the nucleon–nucleon interaction." *M. Cini* (Rome)

2365:

**Schwartz, M. A class of simple field theories and von Neumann's infinite direct product spaces.** Nuovo Cimento (10) **15** (1960), 334–350. (Italian summary)

Quantum mechanical theory differs fundamentally from other branches of mathematical physics in requiring a knowledge of the complete set of states of the physical system under investigation. Even when linear differential equations are used as the basis of the theory it is necessary to formulate the concept of the complete class of solutions to be considered, and local solutions are of no real interest. The most natural mathematical tool is provided by the theory of unitary spaces, since this incorporates the ideas of linearity and orthogonality in one framework.

When this approach is applied to the theory of quantized fields (second quantization) it leads at once to the consideration of infinite products of unitary spaces. The simplest case which is required in a realistic theory is that of a countably infinite product of separable Hilbert spaces. J. von Neumann [Compositio Math. **6** (1938), 1–77] has studied the axiomatic basis for a theory of infinite direct products of unitary spaces. The notions of completeness, convergence, and orthogonality, present major problems. It is only recently that this work has been given consideration by theoretical physicists, as a result of complications which have been encountered in the effort to construct a theory of quantized fields by purely formal methods. A number of relatively simple models have been constructed in which convergence and normalization difficulties can be studied directly. The present paper is an attempt by the author to clarify more completely the connection of the physicists' formal methods with von Neumann's theory.

The case of a scalar boson field in interaction with a fixed point source (nucleon) is chosen for major consideration. By use of box-normalization the first quantization procedure leads only to a countably infinite family of states, each of which becomes a separable Hilbert space under second quantization. The complete set of states is then formed as the countably infinite product of these spaces. The author examines by direct calculation the difficulties encountered in treating this infinite product space as a (non-separable) Hilbert space. The details of the analysis are too intricate to be described here. Troubles arise in defining both normalization and orthogonality. It is pointed out that there exists a whole class of models to which the same type of analysis could be applied. {Unfortunately, quantized field theory contains difficulties which are more profound than those posed in von Neumann's theory. These arise from the attempt to construct the Hilbert spaces considered as the closures of pre-Hilbert spaces derived from the various Hamiltonian operators used in the model. This makes it difficult to define the mathematical basis of physical theory.}

*E. L. Hill* (Minneapolis, Minn.)

2366:

**Abrikosov, A. A.; Gor'kov, L. P.; Dzyaloshinskiĭ, I. E. On the application of quantum-field-theory methods to problems of quantum statistics at finite temperatures.** Ž. Èksper. Teoret. Fiz. **36** (1959), 900–908 (Russian); translated as Soviet Physics. JETP **9**, 636–641.

Matsubara's thermodynamic perturbation theory technique is generalized by considering the Fourier expansion of the Green's function in the imaginary time variable. This makes it possible to work with momentum-space diagrams, which are simply related to the corresponding diagrams for zero temperature, and are almost as easy to evaluate. Exact equations for the Green's functions, analogous to Dyson's equation, are also obtained. It is shown that the time-dependent Green's functions can be obtained by analytic continuation. Thus one can use the methods of quantum field theory to solve various non-equilibrium problems. *T. W. B. Kibble* (London)

2367:

**Laurent, B. E. On a generally covariant quantum theory.** Ark. Fys. **16** (1960), 237–245.

A program for setting up a generally covariant quantum theory is outlined, whose basis is not a particular set of classical fields, but rather the structure of representations of the general coordinate group as discussed by O. Klein [*Niels Bohr and the development of physics*, pp. 96–117, McGraw-Hill, New York, N.Y., 1955; MR **17**, 692]. General properties of these representations are discussed and certain particular fields are derived, which satisfy commutation relations, but not field equations. The dynamics enters through imposition of appropriate subsidiary conditions on the original state space. Fields transforming as second rank tensors are examined in particular, and the structure of subsidiary conditions which would correspond to the Einstein equations is discussed.

*S. Deser* (Waltham, Mass.)

2368:

**Laurent, B. E. Equations for a mixed tensor field.** Ark. Fys. **16** (1960), 247–261.

In connection with the program of the previous paper of the author [see preceding review], the possibility of writing field equations classically without use of the metric field is investigated. This is done by expressing the affinity in terms of a mixed tensor density of weight one half, rather than the usual metric. Finally the relation of this (non-physical) density to the observable metric is made. *S. Deser* (Waltham, Mass.)

2369:

**Finkelstein, David; Misner, Charles W. Some new conservation laws.** Ann. Physics **6** (1959), 230–243.

Authors' summary: "It is shown that field theories possessing a certain type of nonlinearity, termed intrinsic, also possess a new type of conservation law in which the conserved quantity is an integer even in the unquantized theory. For the example of general relativity the conserved quantity is shown to assume the values $M = 0, \pm 1, \pm 2, \cdots$. This conservation law ("conservation of metricity") is valid regardless of any interaction of the metric field with other fields and regardless even of the equation of motion assumed for the metric field itself. The basis of the work is the principle that a quantity which is unchanged in value by an arbitrary continuous deformation is a fortiori unchanged in value by the passage of time. Some properties of metricity and of its carrier are given."

2370:

**Skobelkin, V. I. The propagation of vector waves in nonlinear mesodynamics.** Dokl. Akad. Nauk SSSR **128** (1959), 514–516 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1039–1041.

2371:

**Czerwonko, J. On a generalization of Peierls' theorem.** Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 699–701. (Russian summary, unbound insert)

2372:

**Sugawara, M.; Kanazawa, A. Meson-meson scattering term in pseudoscalar-pseudoscalar meson theory.** Phys. Rev. (2) **115** (1959), 1310–1317.

This paper continues earlier work by the same authors on the static limit of pseudoscalar meson theory. This time a meson-meson scattering term is included in the Hamiltonian and the relevant coupling constants are determined from low-energy meson-nucleon scattering and photon-meson production. *S. Bludman* (Berkeley, Calif.)

2373:

**Steinmann, O. Über den Zusammenhang zwischen den Wightmanfunktionen und den retardierten Kommutatoren.** Helv. Phys. Acta **33** (1960), 257–298. (English summary)

Author's summary: "The connection between Wightman's vacuum expectation values and the retarded functions of Lehmann et al. is investigated in the case of the four point function. Necessary and sufficient conditions for the existence of a Wightman function corresponding to a given $\tau(x_0, \cdots, x_3)$ are derived. The Fourier transform $\tilde{\tau}(p_1, p_2, p_3)$ of $\tau(x_0 - x_1, \cdots, x_0 - x_3)$ is a boundary value of an analytic function $\tilde{\tau}(k_1, k_2, k_3)$, regular in a domain $\mathfrak{D}$ which is defined in the text. Certain boundary values of this function other than $\tilde{\tau}(p_j)$ satisfy a linear identity. This identity enlarges the domain of regularity of $\tilde{\tau}$ still further.

"The Fourier transform $\tilde{\tau}(p_1, p_2, p_3)$ of the time-ordered product $\tau(x_0 - x_1, \cdots) = \langle TA(x_0) \cdots A(x_3) \rangle_0$ is shown to be everywhere a boundary value of the same analytic function $\tilde{\tau}(k_j)$.

"For the $n$-point case it is shown that $W(x_0, \cdots, x_{n-1})$, if it exists at all, is uniquely determined by $\tau(x_0, \cdots, x_{n-1})$ up to terms of a very special kind."

2374:

**Milford, S. N. Approximate cross-sections for inelastic collisions of electrons with atoms. I. Allowed transitions.** Astrophys. J. **131** (1960), 407–412.

Author's summary: "The threshold and large energy properties of cross-sections are combined with the energy derivative of the cross-section at its maximum. The resulting formula relates the maximum cross-section to the energy $\varepsilon_1$ at this maximum. The multipole expansion then gives a simple formula within 10 per cent of the Born approximation maxima for some illustrative transitions of hydrogen induced by electron impact. Choosing $\varepsilon_1$, the approximate formula gives the momentum cutoff in the Bethe approximation. The Bethe and threshold shape is then checked against electron plus hydrogen atom Born cross-sections for allowed transitions. The simple formula gives (a) the Born maximum within a factor of 2 and (b) the Born approximation for all energies within a factor of 3. Thus, when the dipole moment is known from experiment or calculation, the approximate cross-section for any allowed atomic transition can be found at all energies by several minutes' calculation."

2375:

**Pustovalov, G. E. Energy levels and approximate wave functions of mesic atoms.** Ž. Èksper. Teoret. Fiz. **36** (1959), 1806–1817 (Russian); translated as Soviet Physics. JETP **9**, 1288–1295.

Energies and wave functions are given for the first six levels of mesic atoms using a constant charge distribution inside the nucleus. Simple analytical approximations to the wave functions are also given. These are then used in a first-order perturbation method to find relativistic corrections, and to find the shift in levels produced by an exponential charge density distribution.

*D. F. Mayers* (Oxford)

2376:

**Mathur, Vishnu Swarup. Thermodynamic functions of the relativistic Thomas-Fermi atom at low temperatures.** Progr. Theoret. Phys. **23** (1960), 391–399.

Author's summary: "The thermodynamic functions of the relativistic Thomas-Fermi atom, at low temperatures and high pressures, have been obtained in terms of the boundary and initial parameters of the Thomas-Fermi and 'perturbation' equations. Finally these functions are expressed in terms of a parameter $y_0$ which depends on the density of the material. Our final results are, however, valid only when the densities are very high such as those occurring in the interiors of white dwarf stars."

2377:

**Walsh, Peter; Borowitz, Sidney. Application of wave functions containing interelectron coordinates. I. The ground-state energy of lithium.** Phys. Rev. (2) **115** (1959), 1206–1215.

Pluvinage [Ann. Physique (12) **5** (1950), 145–152; MR **12**, 152] has shown how the Schrödinger equation for an atom may be partially separated in such a way that the interelectron potential no longer appears as the perturbing term, and has used this technique in variational calculations on helium-like systems. In this paper, the Pluvinage

method is modified to facilitate variational calculations on larger systems. An approximate calculation for the lithium atom yields satisfactory results.

*A. C. Hurley* (Melbourne)

2378:

**Greider, Kenneth R. Deuteron-pickup reaction in an optical-model approximation.** Phys. Rev. (2) **114** (1959), 786–794.

From the author's summary: "A theory of the $(p, d)$ pickup reaction is described in which the nuclear interactions of the incoming and outgoing particles are considered. Two different formal expressions that give the transition amplitude are derived, and the wave functions in this amplitude are approximated by an optical-model procedure in which it is assumed that the initial- and final-state particles scatter elastically in the nucleus. Several closed forms for these optical-model wave functions, are derived on the basis of a WKB approximation for a complex square-well scattering potential. The use of these wave functions, along with an approximation that gives the form of the transition amplitude in terms of Gaussian functions, allows a closed-form solution for the differential cross section."

*M. L. Goldberger* (Princeton, N.J.)

2379:

**Wapstra, A. H.; Nijgh, G. J.; van Lieshout, R. ★Nuclear spectroscopy tables.** Series in Physics. North-Holland Publishing Co., Amsterdam; Interscience Publishers Inc., New York; 1959. vii + 135 pp. $8.90.

This book was originally planned to be published as an appendix to *Beta- and gamma-ray spectroscopy*, edited by K. Siegbahn [Interscience, New York, 1955]. Different reasons, such as condensing the material and the discovery of the non-conservation of parity, have delayed this program. As it now appears, the book contains tables concerning nuclear spectroscopy, directly or indirectly. The sections describing the method of least squares and quadratic interpolation may fit in a book of this type, but one wonders if this is the place for tables of logarithms, powers of two and ten, etc. Each table starts with a very concise introduction to the subject which gives the proper background. Sometimes these introductions are perhaps a little too short, as e.g. section 7.1, which does not give any definition of electron binding energies, but only a reference. In chapter 2, "Atomic constants", a reference to the recent article in volume 35 of *Handbuch der Physik* [Springer, Berlin, 1957] is obviously missing.

The collection of data given in these tables will no doubt be of great value in the field of nuclear spectroscopy. *P.-O. Lowdin, J.-L. Calais* (Gainsville, Fla.)

2380:

**Fairlie, D. B.; Polkinghorne, J. C. Unstable states and the separable potential model.** Nuclear Phys. **13** (1959), 132–135.

Authors' summary: "A model based on a separable potential is investigated and it is shown that the unstable states can be associated with poles of the continuation of the scattering amplitude into the second Rieman sheet."

*A. Salam* (London)

2381:

**Coulson, C. A. Present state of molecular structure calculations.** Rev. Mod. Phys. **32** (1960), 170–177.

This expository lecture formed the after-dinner speech at the Conference of Molecular Quantum Mechanics, held at the University of Colorado, Boulder, Colorado, June 21–27, 1959; this issue of Reviews of Modern Physics is devoted to papers from this conference.

2382:

**Preuss, H. Die gegenwärtige Situation der Quantenchemie.** Naturwissenschaften **47** (1960), 241–249.

2383:

**Park, David. Relation between the parabolic and spherical eigenfunctions of hydrogen.** Z. Physik **159** (1960), 155–157.

Author's summary: "It is shown that the transformation coefficients relating the eigenfunctions of the Kepler problem in parabolic and spherical coordinates respectively are the normalized Clebsch-Gordan coefficients."

2384:

**Kolos, W.; Roothaan, C. C. J.; Sack, R. A. Ground state of systems of three particles with Coulomb interaction.** Rev. Mod. Phys. **32** (1960), 178–179.

2385:

**McLean, A. D.; Weiss, A.; Yoshimine, M. Configuration interaction in the hydrogen molecule—the ground state.** Rev. Mod. Phys. **32** (1960), 211–218.

2386:

**Frost, Arthur A.; Kellogg, Reid E.; Curtis, Earl C. Local-energy method in electronic energy calculations.** Rev. Mod. Phys. **32** (1960), 313–317.

2387:

**Vanderslice, Joseph T; Mason, Edward A. Quantum mechanical calculations of short-range intermolecular forces.** Rev. Mod. Phys. **32** (1960), 417–421.

2388:

**Osborn, Richard K. Transport in dilute gases and chemical forces.** J. Chem. Phys. **32** (1960), 1817–1820.

Author's summary: "It is shown that the theory of transport phenomena in gases may be so formulated that the potentials characterizing the cross sections, which account for interactions between molecules in the binary collision limit, are the same as those conventionally employed in investigations of molecular structure. Thus it is indicated that at least some of the essential features of chemical forces may be conveniently introduced into the description of such phenomena."

2389:

**Touschek, B. Fixed-source meson theory.** Nuovo Cimento (10) **14** (1959), supplemento, 242–258.

This paper constitutes an excellent elementary review

of the fixed source meson theory formalism: kinematics, the Low equation, and the dispersion relations for pion-nucleon scattering. *S. Bludman* (Berkeley, Calif.)

2390:

**Ashkin, J. Pion-nucleon scattering.** Nuovo Cimento (10) **14** (1959), supplemento, 221–241.

The experimental data on meson-nucleon scattering below 300 Mev is analyzed by assuming charge independence, making the usual phase-shift analysis and comparing with the predictions of the Chew-Low theory and of relativistic dispersion relations.

*S. Bludman* (Berkeley, Calif.)

2391:

**Fano, G. Canonical transformation and perturbation expansion in the theory of Fermi gas.** Nuovo Cimento (10) **15** (1960), 959–969. (Italian summary)

The Bogolyubov-Valatin transformation is used to separate the Hamiltonian of a Fermion system into a 'free quasi-particle' part, and a perturbation term which describes the scattering of the quasi-particles. This perturbation is treated formally with the Goldstone techniques. It does not appear that any new results are obtained. *K. Gottfried* (Cambridge, Mass.)

2392:

**Pais, A. The many $\pi$-meson problem.** Ann. Physics **9** (1960), 548–602.

In view of experimental information obtained in recent years it appears desirable to have methods to describe systems of $N$ mesons in some well organized way that would take the complexity of these multiple-meson states into account. As the multiplicities are rather large and angular momentum, isobasic spin and parity conservation do not lead to sufficiently simplifying restrictions, it seems reasonable to look for correlations of various kinds between the final products, among themselves as well as with the initial particles. The purpose of the present paper is mainly to discuss charge correlations.

It is first shown that the properties of a system of $N$ pions are to a considerable extent determined by three quantum numbers $(N_1, N_2, N_3)$, the "correlation numbers", whose sum equals $N$. In a state with definite correlation these numbers relate to the numbers of $3\pi$-subsystems with isospin $I=0$, the number of $2\pi$-systems with $I=1$, and the remaining single $\pi$-states out of which such a state can be composed by a well defined prescription. The correlation numbers dictate which $I$-spin values a state can possibly have. For $I<2$ the correlation numbers determine $I$ uniquely. Next it is shown that the branching ratios for an $N\pi$-cloud ($I<2$) into the various charge distributions compatible with $N$ and total charge are uniquely determined if one is only given the correlation numbers. General methods are given for determining these branching ratios as functions of the correlation numbers. Applications are given to annihilation processes, with a particular view to the statistical model. All mathematical proofs are collected in a separate section, which contains also some group theoretical work possibly useful in other contexts. The basic tools are Youngs's representations of the symmetric group and Thrall's theorem on its anthonormal idempotents. *P. Roman* (Boston, Mass.)

2393:

**Ito, Daisuke; Minami, Shigeo; Tanaka, Hiroshi. Multipole model of elementary particles. I. Pion-baryon interactions.** Progr. Theoret. Phys. **22** (1959), 159–167.

A simple classical model is described in which strong and weak pion-baryon interactions are interpreted as the monopole and dipole interactions of the pion field with a baryon structure of spatial extension $l$. In such a picture, the distinction between dimensionless and dimensional coupling constants and between renormalizeable and non-renormalizeable interactions is interpreted realistically. The divergences in the nonrenormalizeable interactions at high energies are associated with the failure of the multipole expansion at short distances. To fit the ratio of strong and weak coupling strengths, $l$ must be about $10^{-20}$ cm., which is perhaps surprisingly short for a "fundamental length". *S. Bludman* (Berkeley Calif.)

2394:

**Ito, D.; Minami, S.; Tanaka, H.; Takahashi, Y.; Yamazaki, M. Multipole model of elementary particles. II. Non conservation of parity and baryon-lepton interactions.** Progr. Theoret. Phys. **22** (1959), 168–176.

The discussion reviewed in the preceding abstract is continued to show how parity nonconserving effects will appear in the dipole (weak) interactions if the small-scale baryon structure is, as in optically active molecules, spatially asymmetric. An attempt to apply to $\beta$ decay the model in its crude form leads to the wrong form of interaction and a rather large size ($10^{-14}$ cm.) for the lepton structure. *S. Bludman* (Berkeley, Calif.)

2395:

**Kundu, S. K. Meson production in meson-nucleon interaction.** Progr. Theoret. Phys. **20** (1958), 939–947.

2396:

**Jacob, M.; Wick, G. C. On the general theory of collisions for particles with spin.** Ann. Physics **7** (1959), 404–428.

Authors' summary: "The general analysis of binary reactions involving particles with arbitrary spin is reformulated in such a way that it applies equally well to relativistic particles (including photons). This is achieved by using longitudinal spin components ('helicity states') not only in the initial and final states, but also in the angular momentum states which are employed as usual to reduce the $S$-matrix to a simpler form. Expressions for the scattering and reaction-amplitude, intensity and polarization are given. They involve fewer vector-addition coefficients than the customary formulas, and no recoupling coefficients. The application to some examples is sketched, and in the Appendix some formulas are given that may be of use in the applications."

*M. L. Goldberger* (Princeton, N.J.)

2397:

**Paria, Birendra Nath. Pair production on the basis of hole theory without Dirac matrices.** Indian J. Theoret. Phys. **6** (1958), 69–76.

2398:
**Straka, Josef. Beitrag zur Algebra des isobarischen Spins.** Apl. Mat. **5** (1960), 63–71. (Czech. Russian and German summaries)

2399:
**Aeschlimann, Florence. Onde moyenne d'un système de corpuscules en théorie fonctionnelle.** J. Phys. Radium **20** (1959), 927–932. (English summary)

Author's summary: "Some principles of the functional theory of particles and the properties of the centre of masses are discussed. The definitions of mean-waves and sum-wave are given, in particular the mass-mean-wave (which corresponds to the centre of masses) and the uniform-mean-wave. The global properties of a system can be expressed with these waves."

2400:
**Tzou, K. H. Les groupes de transformation et les états propres dans les théories du neutrino.** J. Phys. Radium **21** (1960), 537–543. (English summary)

Author's summary: "In the case of a particle of spin 1/2 and of zero rest mass, we identify the proper states generated by the inversion group $\mathcal{G}$ in the four-component as well as in the two-component theory. The constant of motion $\gamma_5$, chirality, which is intimately connected to gauge invariance $M$, is taken into consideration. On the other hand, owing to an arbitrariness in the choice of the definition of solutions in the case of zero rest mass, charge conjugation leads to gauge invariance $C$, when a neutral particle is considered. It is shown that gauge invariance $M$ is a perfect invariance, but gauge invariance $C$ is not, if one takes into account angular momentum, helicity and chirality quantum numbers."

2401
**Królikowski, W. On the vector bosons.** Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 729–731. (Russian summary, unbound insert)

2402:
**Królikowski, W. Effective range approximation for scattering of kaons with scalar coupling.** Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 695–697. (Russian summary, unbound insert)

2403:
**Królikowski, W. A fixed-source approach to scattering of kaons.** Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 237–244; supplement, 691–693. (Russian summary, unbound insert)

2404:
**Cini, M. Nucleon structure.** Nuovo Cimento (10) **14** (1959), supplemento, 408–414.

2405:
**Brodskiĭ, A. M.; Ivanenko, D. D. Anomalous spinors and bosons.** Ž. Èksper. Teoret. Fiz. **36** (1959), 1279–1285 (Russian); translated as Soviet Physics. JETP **9**, 907–911.

Anomalous spinors are those for which the operations of space and time reflection commute, whereas these operations anticommute for normal spinors. The authors claim that for anomalous spinors (and for anomalous bosons formed by fusion), the Lüders PCT theorem and the normal connection between spin and statistics need not hold. They recommend assigning leptons to the normal, and baryons to the anomalous, representations.

*S. Bludman* (Berkeley, Calif.)

2406:
**Lundqvist, S. O. Cluster development of the density matrices in the Jastrow method for many-fermion systems.** Ark. Fys. **16** (1960), 321–328.

Author's summary: "A method is given for the cluster development of the reduced density matrices for Jastrow wave functions. The essence of the method is to make extensive use of the simple properties of the Dirac density matrices corresponding to the uncorrelated motion, in conjunction with the method of expanding the distribution functions for a classical imperfect gas given by Mayer and Montroll." *K. Gottfried* (Cambridge, Mass.)

2407:
**Pétermann, A.; Ruegg, Henri. Groupe mésonique et conservation de la parité.** Helv. Phys. Acta. **33** (1960), 143–160. (English summary)

Authors' summary: A principle of invariance under a continuous local group of transformations, the mesic group, is being investigated. This principle has the following consequences. (1) For the pseudoscalar Yukawa interaction of two Fermions with the pseudoscalar $\pi$-meson it entails $PC$ invariance. (2) If the Fermions have equal bare masses with respect to electromagnetic interaction (a hypothesis which is plausible for the nucleons), the principle imposes, for the $ps$ interaction with $\pi$, the conservation of isotopic spin and separate $P$ and $C$ invariance. (3) For the Fermi interactions of the pairs $(pn)$, $(\nu\mu^-)$, $(\nu e^-)$, etc., it involves $V$ and $A$ coupling, with non-conservation of parity. Our arguments leading to this principle are based on a generalization of the demonstration of the Dyson-Foldy equivalence theorem as given by Stueckelberg, and one of us."

2408:
**Misra, S. P. Fourth order scattering matrix elements of nucleons with a fourth order meson equation.** Indian J. Phys. **34** (1960), 221–236.

2409:
**Theis, W. R. Zweikomponentige Spinorfelder zur Beschreibung beliebiger Teilchen vom Spin 1/2 und ihre Anwendung auf die schwachen Wechselwirkungen.** Fortschr. Physik **7** (1959), 559–583.

This is a thorough-going review paper which develops the relativistic field theory of spin one-half particles, employing exclusively two component field operators which satisfy, in the absence of interactions, the Klein-Gordon equation, and transform according to the spinor representation of the proper homogeneous Lorentz group. The main

topics discussed include the quantization of non-interacting fields, coupling to an electromagnetic field, and construction of four-fermion interactions.

*A. Klein* (Philadelphia, Pa.)

2410:

**Cabibbo, N.; Gatto, R. Structure of weak interactions and unwanted processes.** Phys. Rev. (2) **116** (1959), 1334–1338.

Authors' summary: "A general discussion of the forbidden reactions such as $\mu \to e+\gamma$, $\mu \to e+e+e$, $\mu+N \to e+N$, $\mu \to e+\gamma+\gamma$, etc., which would arise from a possible nonlocal structure of the weak interactions is given. It is shown, by a canonical transformation, that the dominating terms of the structure, in an expansion in terms of the inverse of the average intermediate mass, do not contribute to any forbidden reaction. A discussion of the $\mu-e$ conversion process for a bound $\mu$ is given, and the rate is calculated for a particular model."

*S. Bludman* (Berkeley, Calif.)

2411:

**Moffat, John W. On the boson mass renormalization constants in pseudoscalar meson theory.** Nuclear Phys. **13** (1959), 150–155.

Author's summary: "The purpose of this paper is to investigate the significance of Lehmann's theorem, which states that for pseudoscalar mesons $\delta m^2 \leqq 0$. This theorem prevents us from ascribing the meson mass to self-mass effects alone. By including a $\lambda\phi^4$ term in the Lagrangian it is proved that Lehmann's theorem is no longer necessarily fulfilled. On the basis of this conclusion one can no longer exclude the possibility that the mass of the meson in the absence of interaction is zero, and that the $\phi^4$ coupling produces most of the observable meson mass. However, in view of the singular behaviour of the modified propagator, the vacuum expectation value $\langle\phi^2\rangle_0$ is divergent and $\delta m^2$ appears to be either infinite or indeterminate."

*A. Salam* (London)

2412:

**Tzou, K. H. Normalisation uniforme des ondes planes de spin 1/2 sous les opérations d'inversion.** J. Phys. Radium **21** (1960), 579–586. (English summary)

Author's summary: "The possibility of a uniform normalization of monochromatic plane waves is examined with respect to inversion operations. It is shown that such a normalization is realizable for each of the eight unitary inversions, but not for the anti-unitary ones, except $C$ and $PTM$. A uniform normalization is even possible for many inversions simultaneously, in particular for the four operations $CPT$, $M$, $CT$, $PM$. Owing to the existence of uniform normalization of plane waves with respect to unitary inversions, we find finally spinor field operators which are invariant under these inversions, especially under $CPT$, $M$, $CT$, $PM$."

2413:

**Cutkosky, R. E. Radiative meson-nucleon scattering.** Phys. Rev. (2) **109** (1958), 209–217.

2414:

**Bosco, B.; Stroffolini, R. The radiative pion-nucleon scattering in static theory.** Nuovo Cimento (10) **10** (1958), 688–697. (Italian summary)

Pion-nucleon scattering with an additional photon emitted in the final state is discussed in the static approximation. The treatment differs from that of Cutkovsky [#2413 above] in that the two pions are not treated symmetrically and hence it is easier to discuss the case when one pion is in the 3,3 resonance state. The treatment parallels closely that of G. F. Chew and F. E. Low [Phys. Rev. (2) **101** (1955), 1579–1587] of ordinary pion-nucleon scattering. The calculations lead to an integral equation. In view of the fact that the rescattering (enhancement) is most important in the 3,3 state, the equations are considered for the magnetic dipole and electric quadrupole amplitudes. The solution was given by Omnès [Nuovo Cimento (10) **8** (1958), 316–326; MR **20** #688].

*M. J. Moravcsik* (Livermore, Calif.)

2415:

**Brown, G. E.; DeDominicis, C. T.; Langer, J. S. Perturbation theory in nuclear reactions.** Ann. Physics **6** (1959), 209–229.

Authors' summary: "Exact expressions for the amplitudes for scattering of a particle by a complex nucleus are written down. It is then shown that, with a particular weight function, the scattering amplitude can be averaged over energy by going to a complex energy, i.e., $[S(E)]_{Av} = S(E+iI)$, where $I$ is the interval averaged over.

"The average amplitude is then expressed in terms of a perturbation expansion. In perturbation theory of the first kind, expansion in powers of the interaction potential between the incident particle and the particles in the nucleus is carried out. In the second kind of perturbation theory, all particles are treated symmetrically and all but the average effects of the interactions are treated as perturbations. This allows one to relate the parameters of the optical potential back to nucleon–nucleon forces.

"It is shown that these expansions are, in general, convergent, due to the fact that the excitation into which a given excitation decays has a longer life-time than the original one."

*M. Cini* (Rome)

## RELATIVITY

See also 2274, 2275, 2276.

2416:

**Kalicin, Nikola St. On the influence of the rotation of a body of spherical symmetry on the displacement of the perihelion of its satellites according to Einstein's theory of gravitation in second approximation.** Izv. Bŭlgar. Akad. Nauk. Otd. Fiz.-Mat. Tehn. Nauk. Ser. Fiz. **7** (1959), 319–331. (Bulgarian. Russian and English summaries)

The author shows that the first order approximation formula of Lense and Thirring [Phys. Z. **19** (1918), 156–163] for the perihelion displacement continues to hold when second order terms in the velocity ratio are retained.

*R. G. Langebartel* (Urbana, Ill.)

2417:

**Kar, K. C. Note on the linearisation of the relativistic Hamiltonian.** Indian J. Theoret. Phys. **6** (1958), 107–109.

2418:

**Sauter, Fritz. Zur Lorentz-invarianten Formulierung der kanonischen Bewegungsgleichungen in der Punktmechanik.** Z. Physik **156** (1959), 275–286.

With the help of the Hamiltonian variational principle, the equations of motion of a particle in special relativity are put into a form which is covariant and resembles that of the classical canonical equations of motion. The equations contain two scalar functions of the coordinates and momenta, connected by one relation, expressing the constancy of the magnitude of the four-velocity. Special forms of these equations, in which one of these functions is equal to unity, have been given by G. Falk [Z. Physik **132** (1952), 44–53; MR **14**, 1045] and W. Macke [Z. Naturforschg. **7a** (1952), 76–78; MR **14**, 98]. Corresponding to the present equations, the invariant Hamilton-Jacobi equation is derived. *N. Rosen* (Haifa)

2419:

**Bolinder, E. Folke. Impedance, power, and noise transformations by means of the Minkowski model of Lorentz space.** Ericsson Technics **15** (1959), 249–283.

Author's summary: "It is shown how the Minkowski models of two-, three-, and four-dimensional Lorentz space can be used geometrically in transforming impedance, power, and noise quantities through bilateral two-port networks. The transformations constitute direct generalizations of some earlier geometrical methods that have been applied in the complex impedance plane, the complex reflection-coefficient plane, and the Cayley-Klein models of two- and three-dimensional hyperbolic space.

"The geometric operations performed in the Minkowski models of the Lorentz spaces are put into analytic form by means of matrix algebra. This standard treatment is compared with a treatment by means of an associative, but noncommutative, algebra called 'Clifford algebra'. This elegant and powerful mathematical tool is briefly outlined.

"The application of the geometric-analytic theory in network theory is illustrated by means of several numerical examples." *R. Kahal* (Washington, D.C.)

2420:

**Harrison, B. Kent. Exact three-variable solutions of the field equations of general relativity.** Phys. Rev. (2) **116** (1959), 1285–1296.

The author investigates the solutions of the equations of general relativity for empty space under the restrictive condition that the space time admits a coordinate system in which the metric tensor assumes the form:

$$g_{ik} = \delta_{ik} e_i A_i^2(x^0, x^1) B_i^2(x^0, x^3),$$

where $e_0 = -1$, $e_1 = e_2 = e_3 = +1$ and the functions $A_i$ and $B_i$ are determined by the field equations. A fairly large number of solutions are presented although their physical interpretation remains rather obscure.

*A. Raychaudhuri* (Calcutta)

2421:

**Pham Tan Hoang. Sur les équations du mouvement en relativité générale.** C. R. Acad. Sci. Paris **250** (1960), 1195–1197.

The equations of motion for two particles follow from the equation of the "geodesic" line. *L. Infeld* (Warsaw)

2422:

**Le-Thanh-Phong. Vecteur de Poynting en relativité générale.** C. R. Acad. Sci. Paris **250** (1960), 987–989.

It is shown that the 'Poynting vector' of gravitational radiation, defined, in a natural way, in terms of the Bel-Robinson tensor [L. Bel, same C. R. **247** (1958), 1094–1096; MR **20** #6307] is identical with that defined in terms of the canonical energy pseudo-tensor, in normal coordinates, by the reviewer [Phys. Rev. (2) **105** (1957), 1089–1099; MR **20** #3020]. *F. A. E. Pirani* (London)

2423:

**Kremer, Hugo; Kichenassamy, S. Sur le champ électromagnétique singulier dans une théorie du type Born-Infeld.** C. R. Acad. Sci. Paris **250** (1960), 1192–1194.

The results of the Born-Infeld theory are investigated under general assumptions: $L$ (the Lagrangian) depends on two invariants $F = \frac{1}{4}\varphi_{\mu\nu}\varphi^{\mu\nu}$ and $G = \frac{1}{4}\varphi_{\mu\nu}\varphi^{*\mu\nu}$. The metrical form is $ds^2 = j_{\alpha\beta}dx^\alpha dx^\beta$. *L. Infeld* (Warsaw)

2424:

**Laurent, B. E. A variational principle and conservation theorems in connexion with the generally relativistic Dirac equation.** Ark. Fys. **16** (1960), 263–278.

The coupled gravitation-Dirac fields are examined using variables introduced by O. Klein [Ark. Fys. **16** (1959), 191–196; MR **21** #7763]. These quantities are the $\gamma^\mu$ and $\Gamma_r$ appearing in the generally covariant Dirac equation. By varying them independently (with a suitable constraint on the variations of $\gamma^\mu$) one obtains directly the usual Einstein equations. This method is then used to derive conservation laws for the gravitational field, and in particular that of Møller [Ann. Physics **4** (1958), 347–371; MR **20** #732]. *S. Deser* (Waltham, Mass.)

2425:

**Laurent, B. E. Remarks on the functional integration quantization of gravitation.** Ark. Fys. **16** (1960), 279–283.

The following aspects of functional integral (Feynman) quantization of the gravitational field are considered. (1) A possible weighting to be used in the integration over metrics is derived on group theoretical grounds. (2) The question of handling integrations over different "gauges" —(i.e., coordinate systems) which express the same physical history is treated. On the assumption that the quantities involved are "gauge" invariant, the full integration is related to that keeping within one gauge, since both sum over all physically different histories. Some of the ambiguities of this quantization in relativity are indicated. *S. Deser* (Waltham, Mass.)

2426:

**Rayner, Charles Beresford. Sur une simplification des equations exterieures d'Einstein pour un mouvement de groupe.** C. R. Acad. Sci. Paris **249** (1959), 1614–1616.

The author considers the classical relativistic equations of Einstein for empty space. When there exists a time-like vector which satisfies the equations of Killing, it is possible to reduce the solution of the equation $R_{ij} = 0$ to a corresponding problem in three dimensions.

*M. Wyman* (Edmonton, Alta.)

2427:

**Rayner, Charles Beresford. Une forme simple pour le tenseur de Ricci sous conditions de rigidité.** C. R. Acad. Sci. Paris **249** (1959), 1461–1463.

The author derives a simple form for the Ricci tensor when the equations of rigidity of Rosen are satisfied by a unitary vector. *M. Wyman* (Edmonton, Alta.)

2428:

**Rayner, Charles Beresford. Trois remarques concernant un théorème récent.** C. R. Acad. Sci. Paris **249** (1959), 1327–1328.

This paper clarifies the roles played by certain functions and equations in the proof of a theorem contained in a recent paper [Rayner, same C. R. **248** (1959), 929–932; MR **21** #1200]. *M. Wyman* (Edmonton, Alta.)

2429:

**Takeno, Hyôitirô. On geometric properties of some plane wave solutions in general relativity. II.** Tensor (N.S.) **9** (1959), 162–174.

The present paper is a continuation of part I [Tensor (N.S.) **9** (1959), 76–93; MR **21** #5481] by the same author. If either the pseudo-tensor of Einstein or that of Landau and Lifshitz is used to express the pseudo-tensor of energy-momentum, it is shown that there exist gravitational waves that carry energy and momentum in the direction of propagation.

In the latter part of the paper, the author studies in detail the eigen bivectors of the curvature tensor and gives both geometric and physical interpretations of his result. *M. Wyman* (Edmonton, Alta.)

2430:

**Shibata, Takashi. Geometrical definition of plane gravitational waves in general relativity.** Mem. Fac. Engrg. Hiroshima Univ. **1**, 165–184 (1960).

The author gives a detailed geometrical discussion of the plane gravitational waves discovered by Bondi, Pirani and Robinson [Proc. Roy. Soc. London Ser. A **251** (1959), 519–533; MR **21** #5478] and by Takeno [see preceding review]. *D. W. Sciama* (Ithaca, N.Y.)

2431:

**Costa de Beauregard, Olivier. Développement des conséquences de la théorie de l'inertie de D. W. Sciama et de D. Park.** C. R. Acad. Sci. Paris **250** (1960), 2149–2151.

2432:

**Wagh, Ramesh V. Gravitational field of distant rotating masses.** Indian J. Phys. **34** (1960), 211–216.

Author's summary: "In 1958, Thirring calculated the gravitational field near the centre of a rotating spherical shell. The case can be generalised to that of a rotating mass, where the field away from the mass can be determined. Thirring assumed $T^{\mu\nu}=\rho v^{\mu\nu}$. But it can be shown that, starting from a Galilean field we can build up the case of a non-Galilean field by introducing some small term in the metric tensor whose Galilean value is unity. Then by a straightforward process, we calculate $T^{\mu\nu}$ given by $T^{\mu\nu}=(p_0+\rho_0)v^\mu v^\nu-p_0g^{\mu\nu}$. By the introduction of a rotating mass in the Galilean field, which now slightly deviates from its original characteristics, the metric tensor is given by $g_{11}$, $g_{22}$, $g_{33}=-(1+\alpha)(1, r^2, r^2\sin^2\theta)$ and $g_{44}=(1-\alpha)$. The energy-momentum tensor is then calculated with $\alpha=m(1-r^2\sin^2\theta\omega^2)$ and it is shown that this satisfies the conditions of mechanics. It is incidentally shown that there is no necessity of introducing $E^{\mu\nu}$ in the expression for $T^{\mu\nu}$, as recently done by Bass and Pirani [Phil. Mag. (7) **46** (1955), 850–856; MR **18**, 704]. The desired results are obtained without making such assumptions."

2433:

**Costa de Beauregard, Olivier. Métrique asymétrique et représentation des changements d'axes locaux. Application à la théorie de l'effet gravitationnel de spin.** C. R. Acad. Sci. Paris **250** (1960), 984–986.

The author defends his proposed equations of motion [C. R. Acad. Sci. Paris **247** (1958), 1092–1094; MR **20** #7577] for a spinless particle in the neighborhood of a spinning particle against the criticism of the reviewer [MR **20** #7577]. *D. W. Sciama* (Ithaca, N.Y.)

2434:

**Lenoir, Marcel. Sur les solutions à symétrie sphérique de la théorie du champ unifié.** C. R. Acad. Sci. Paris **250** (1960), 981–983.

The author considers the non-symmetric unified field theory in one of its original forms, and also in the modified form given by Bonnor. He obtains spherically symmetric solutions for both forms by a variational principle. The derivation of an exact solution for Bonnor's theory appears to remove doubts about the compatibility of the equations of this theory, raised in a previous paper by the author [same C. R. **249** (1959), 44–46; MR **21** #6259].

*W. B. Bonnor* (Urbana, Ill.)

2435:

**Droz-Vincent, Philippe. Une méthode de quantification en théorie unitaire pentadimensionnelle.** C. R. Acad. Sci. Paris **248** (1959), 1790–1792.

2436:

**Venini, Carlo. Moto di dipoli elettrici nell'ultima teoria unitaria einsteiniana.** Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **26** (1959), 490–497.

Einstein, Infeld and Hoffmann [Ann. of Math. (2) **39** (1938), 65–100] determined from the field equations for empty space the equations of motion of matter represented as point singularities of the field. Similar methods were applied with regard to charged particles in the so-called unified field theories to a first approximation by Clauser [same Atti (8) **21** (1956), 408–416; MR **19**, 1021] and by Treder [Ann. Physik (6) **19** (1956), 369–380; MR **19**, 816]. In the present note the techniques of Clauser are applied to the determination of the motion of charged material particles which are endowed also with an electric dipole. The equations of motion which are obtained to a first approximation contain not only the usual mutual interaction forces, but additional forces in the direction of preferred axes associated with each particle.

*H. Rund* (Durban)

2437:

**Surin, Aline. Sur la méthode des singularités en théorie pentadimensionnelle de Jordan-Thiry.** C. R. Acad. Sci. Paris **249** (1959), 2279–2281.

Calcul des équations du mouvement en première approximation dans la théorie unitaire de Jordan-Thiry par la méthode des singularités. En modifiant l'ordre des premiers termes des développements de $\gamma_{04}$ et $\gamma_{04}$ adopté par F. Hennequin [F. Hennequin, thèse, Gauthiers-Villars 1956] dans la méthode du tenseur d'impulsion énergie, l'A. trouve coincidence des résultats par les deux méthodes.

*Y. Fourès-Bruhat* (Paris)

2438:

**Takasu, Tsurusaburo. Adjusted relativity theory: applications of extended Euclidean geometry, extended equiform geometry and extended Laguerre geometry to physics.** Yokohama Math. J. **7** (1959), 1–41.

## ASTRONOMY

See also 2064, 2465.

2439:

**Eddington, A. S. ★The internal constitution of the stars. With a new introduction by Lloyd Motz.** Dover Publications, Inc., New York, 1960. xvi + 407 pp. Paperbound: $2.25.

I am extremely glad to have a new copy of *The Internal constitution of the stars*, not because I want to throw away my tattered first edition but because I would like to preserve it carefully, and now I can refer for working use to the new paper-back copy. Eddington's book is a real classic, and whenever one looks at the text to see exactly what he says about anything one is impressed by his wisdom and by the fact that he almost always says something much nearer our contemporary views than one can imagine anyone writing in the 1920's having a right to say. It may be that younger scientists who are trained as undergraduates in the quantum theory may feel impatient at the hesitations of the pioneers grouping towards it; one hopes that they won't. One certainly hopes that no scientist who reads Eddington will fail to be charmed by his liveliness and by his lucidity.

In this and in several other books Eddington keeps in close touch with the facts unearthed by the observers. He was himself at one time a Greenwich man and an observer, and in *Internal constitution* he chose to keep both feet on the ground, not because he was pedestrian and afraid to fly, but because he was convinced of the necessity of staying close to the observed facts about the stars, when it was stars he was writing about.

One may perhaps regret Eddington's insistence on his polytropic stars as physical entities rather than as mathematical conveniences, and his skating over the point that although he could go very far without knowing the laws of energy generation, his successors would go much further if and when they did; but as the full contemporary treatment of stellar interiors demands the calculating machine, and the simpler treatment is able to dispense with everything except Emden's tables and can therefore show one compact result, it is no doubt still necessary to master Eddington's treatment before attempting to understand what is done nowadays. I for one applaud the Editors of this series for bringing out new editions of important works which are now hard to come by, in the belief that their perusal is of the greatest use to contemporary students; and I am quite convinced that the astrophysicist of the latter half of this century has no better model than Sir Arthur Eddington.

*R. v. d. R. Woolley* (Sussex)

2440:

**Mikhailov, A. A. The deflection of light by the gravitational field of the sun.** Monthly Not. Roy. Astr. Soc. **119** (1959), 593–608.

Expository lecture.

2441:

**Meffroy, Jean. Partie périodique de la fonction perturbatrice et nouveau terme séculaire pur de la perturbation du troisième ordre des grands axes.** Bull. Astr. **23** (1960), 149–185. (English, German and Russian summaries)

Author's summary: "As Tisserand dimly perceived, the purely secular term of the third order perturbation of the major axes originates, on the one hand from the secular part, and on the other hand from the periodic part of the disturbing function. In a previous paper, we computed the purely secular term which originates from the secular part of the disturbing function. The purely secular term which originates from the periodic part of the disturbing function is computed here, using the same hypothesis."

2442:

**Message, P. J. Some periodic orbits in the restricted problem of three bodies and their stabilities.** Astr. J. **64** (1959), 226–236.

From the author's summary: "Periodic orbits in the restricted problem of three bodies near the exterior case of 2:1 commensurability of period have been found by step-by-step numerical integration, using the IBM 650 computer at the Yale University Computing Center. Periodic orbits with eccentricities up to 0.4 have been traced on the two series of symmetric orbits and also on one of the series of asymmetric orbits, and on one series of symmetric orbits members were traced with eccentricities up to 0.9. For the orbits of all but the largest eccentricities the coordinates are exhibited as Fourier series. The ordinary stabilities of a selection of the orbits of eccentricities up to 0.1 were investigated using a method deriving from Brown's method for the determination of the motion of the lunar perigee. The symmetric orbits of small eccentricity prove to be stable, those of larger eccentricity unstable, and the asymmetric orbits investigated are stable, there being an exchange of stability at the point of bifurcation."

*E. Leimanis* (Vancouver, B.C.)

2443:

**Kaminisi, Keisuke. On the degree of electron degeneracy in the stellar interior.** Kumamoto J. Sci. Ser. A **4**, 11–15 (1959).

Author's summary: "The degree of electron degeneracy in the stellar interior decreases outward in the region where the transfer of energy is governed by a radiative or conductive process and is constant in the region where the transfer of energy is governed by a convective process."

2444:

**Huang, Su-Shu. A perturbation method in the theory of stellar structure.** Astrophys. J. **131** (1960), 452–458.

The author develops a method for calculating the perturbations of the eigenvalues appearing in the theory of stellar structure. If a certain model of a star has been computed, its neighboring configurations can be computed easily by this method.

This method has been applied to the stellar model consisting of a radiative envelope over a convective core.

*Y. Kozai* (Cambridge, Mass.)

2445:

**Thomas, Richard N. The source function in a non-equilibrium atmosphere. IV. Evaluation and application of the Net Radiative Bracket.** Astrophys. J. **131** (1960), 429–437.

Author's summary: "The net rate of a radiative transition between two atomic energy levels depends upon the local radiation field through a factor which we define as the Net Radiative Bracket (NRB). The results of previous papers in this series are applied to show that NRB depends only upon the ratio of radiation absorbed in the line to the local value of the source function in the line and to evaluate this ratio as a function of optical depth in the line. These results may also be applied to the question of the local energy balance in a chromosphere-type atmosphere and to the problem of radiative stability."

2446:

**Chandrasekhar, S. ★Radiative transfer.** Dover Publications, Inc., New York, 1960. xiv+393 pp. Paperbound: $2.25.

This is an unabridged and slightly revised version of the work first published in 1950 [Oxford Univ. Press, New York] and reviewed in MR **13**, 136.

2447:

**Ramamoorthy, P.; Chakraborty, B. B. A note on force-free fields.** Proc. Nat. Inst. Sci. India. Part A **25** (1959), 388–393.

Authors' summary: "We consider a self-gravitating cylindrical mass of compressible fluid having infinite conductivity. Initially it is supposed to be in equilibrium under the action of its gravity, an axi-symmetrical magnetic field and an external gas pressure. The purpose of the paper is to find out the displacements $\xi=\{0, 0, \zeta(\bar{\omega}, \phi)\}$ which would render the magnetic field force-free preserving the hydrostatic equilibrium of the mass. We also calculate the corresponding changes in the magnetic energy per unit length of the cylinder and the pressure distributions in two particular cases."

2448a:

**Kaplan, S. A.; Klimišin, I. A. Shock waves in stellar envelopes.** Astr. Ž. **36** (1959), 410–421. (Russian. English summary)

2448b:

**Kaplan, S. A.; Klimishin, I. A. Shock waves in stellar envelopes.** Soviet Astr. AJ **3**, 404–414 (1959).

{The second article is a translation of the first.}

There is in general no exact solution to the problem of the propagation of a shock wave in a stellar atmosphere; consequently each worker makes the approximations which seem most suited to the problem in hand. The novel feature of the present paper is that the authors manage to discuss the separation of the outer envelope of a red giant star without introducing the equations of gas dynamics.

The authors assume that, at each (Eulerian) radius, the temperature behind the shock front is essentially the same as in the final equilibrium state which is established long after the shock has passed. This is tantamount to asserting that when the gases are brought up from the lower (hotter) layers they always expand by just the right amount to keep the temperature constant at each (given) radial distance. The law of shock propagation is then readily inferred, and it is suggested that the envelope separates where the shock communicates the escape velocity to the gas. This will at most only be approximately true, since the gases are further accelerated by the rarefaction wave associated with the shock. Allowing for the heat exchange at the shock due to recombination, the authors conclude that separation of mass in the manner compatible with the above assumption is only possible for red giants ($M\sim M_\odot$, $R\sim 100R_\odot$) and it then takes place at one half the stellar radius.

When a shock front approaches a stellar surface, it loses radiation directly into space; the authors find this effect can be important even at large optical depths. The paper concludes with a discussion of the effect, on the shocked gases, of degeneracy in the quiet medium.

*J. Hazlehurst* (Chicago)

2449:

**Kustaanheimo, Paul. Time derivatives of the components of proper motions of stars.** Astr. J. **65** (1960), 46–47.

Author's summary: "The rates of change with time in the spherical components of the constant motion of a star are deduced by means of direct projection on the local coordinate base vectors. In many textbooks on spherical astronomy these formulae are given incorrectly."

*J. A. O'Keefe* (Chevy Chase, Md.)

2450:

**Just, Kurt. Zur Wechselwirkung zwischen Nebelhaufen.** Z. Astrophys. **49** (1960), 19–24. (English summary)

The author takes the metric

$$ds^2 = \xi^2 dr^2 + \eta^2(d\theta^2 + \sin^2\theta d\phi^2) - dt^2,$$

where $\xi$, $\eta$ are functions of $r$ and $t$, to represent a spherically symmetric model of the universe containing a continuous distribution of incoherent matter. He then shows that if at $t=t_0$ the model is homogeneous outside a condensation $r\leq r_0$, it will remain so for all time. He then considers a similar model universe with several condensations, and shows that in this case too, if the model is homogeneous outside the condensations, it will remain so; further, each condensation is unaffected by the presence of the others.

The conclusion is that spherically symmetric clusters of galaxies partake of the expansion of the universe, and separate clusters have no influence on each other. {Reviewer's remark: The author shows that non-overlapping inhomogeneous regions imbedded in a uniform world model do not interact. To apply this conclusion to clusters of galaxies, it is necessary to show, using the author's

model, what kind of clusters can be represented by such non-overlapping regions. Further work is required to investigate this.} *W. B. Bonnor* (Urbana, Ill.)

2451:

**Dufour, Henri-Marcel; Fontaine, André. Formules pratiques pour le calcul électronique des coordonnées des étoiles.** Bull. Astr. **23** (1960), 117–126. (English, German and Russian summaries)

Authors' summary: "The authors expose a method—already experimented at the Institut Géographique National—for computing, on electronic machines, the various kinds of star coordinates used in Astronomy of position. They recommend the use of ecliptic coordinates and transformations of rectangular coordinates, using closed formulae: this leads to very general working methods, using only one reference card catalogue, and programmes very easy to write for electronic computation."

2452:

**Dougherty, J. P. A statistical theory of ionospheric drifts.** Phil. Mag. (8) **5** (1960), 553–570.

Author's summary: "An attempt is made to interpret the horizontal 'drift velocities' observed in ionospheric fading-wave experiments by examining statistically the fluctuations in space and time of the density of ions and electrons. The ionization is assumed to be convected by a turbulent flow in the neutral air, but the magnetic field and the dynamo electric field are also taken into account. The drift velocity to be calculated is defined by means of the space-time correlation function.

"For the $D$ region and lower $E$ region, the estimation of this drift velocity is found to be mathematically practicable, using certain assumptions. Results are obtained which show the relative importance of the mean aerodynamic velocity and of the electric field in determining the drift of irregularities in electron density.

"For higher regions, the theoretical problem is much less tractable, for reasons discussed here; but it is pointed out that in this case it is unlikely that the irregular behaviour can be ascribed to turbulence anyway, and that the problem of the drift velocity cannot properly be investigated until a satisfactory explanation of the cause of the irregularities is available.

"Some remarks are offered on the relation of this theory to the observations and to the dynamo theory."

## GEOPHYSICS

See also 1960, 2029, 2228, 2229, 2452.

2453:

**United States Air Force, Geophysics Research Directorate. ★Handbook of geophysics.** Revised ed. The Macmillan Co., New York, 1960. xvii+656 pp. $15.00.

This book is a revised and extended version of a book *Handbook of geophysics for Air Force designers* [U.S. Air Force, Cambridge, Mass.] published in 1957 for the use of air force contractors to help them in the planning and design of the multitude of devices and structures scattered throughout the world and in the atmosphere that the Air Force needs to have either manufactured or constructed. It was written by about 57 different authors and gives numerical data and design curves that describe the atmosphere, the surface of the oceans, and the soil and permafrost in the upper layers of the earth's crust.

There are twenty-one chapters and over 640 pages on subjects such as model atmospheres, wind, geomagnetism, terrestrial surface parameters, the sun, atmospheric exploratory devices, and acoustic propagation in the atmosphere.

The book is a source of valuable data on the earth and the atmosphere. *W. J. Pierson, Jr.* (New York)

2454:

**Jensen, Arne. Problèmes de probabilité et décision en connexion avec les raz de marée.** Le calcul des probabilités et ses applications. Paris, 15–20 juillet 1958, pp. 89–101. Colloques Internationaux du Centre National de la Recherche Scientifique, LXXXVII. Centre National de la Recherche Scientifique, Paris, 1959. 196 pp.

The usual assumption of an exponential distribution for the height of tides $x$ is refuted by the comparison of the means and variances observed on four stations of the Danish North Atlantic coast during the last 80 years. Instead the author uses one member of Pearson's Type I and estimates the two parameters by the method of moments. The assumption that the number of annual storms has a Poisson distribution along the whole coast with a mean $N$ (about 5) leads to the probability for the largest height to remain below $x$ within $T$ years. The mean and variances of the $m$th heights $x_m$ ($m$ counted from above) for $NT$ observations and of their differences are obtained from Cramer's transformation $\xi_m = NT[1 - F(x_m)]$. Such estimations are necessary for construction of dikes which involve the danger that the past has been too benevolent and future tides surpass the dike. The cost of such a catastrophe within a given period has to be evaluated against the cost of enlarging the dike. Following van Dantzig's procedure [Econometrica **24** (1956), 276–287; MR **19**, 230] he constructs a decision rule for the height of the dike which involves the estimation of the upper limit of the height of the tides. This estimation remained unclear to the reviewer.

The paper would have gained by numerical or graphical comparisons of the initial and largest observations to the theories and the forecast derived therefrom.

*E. J. Gumbel* (New York)

2455:

**Pichler, H; Reuter, H. Zur graphischen Integration der Wirbelgleichung mit Berücksichtigung orographischer Effekte.** Arch. Meterol. Geophys. Bioklimatol. Ser. A **11**, 413–426 (1960). (English and French summaries)

2456:

**Hess, Seymour L. ★Introduction to theoretical meteorology.** Henry Holt and Co., New York, 1959. xiv+362 pp. $8.50.

During the past two decades theoretical meteorology has made great strides and many of the textbooks in existence today have understandably failed to present an adequate overall account of the subject. It is therefore

refreshing to encounter a book which, as well as setting out clearly the underlying physical principles, gives a balanced review of the subject as a whole.

The demands made upon the reader are limited to a knowledge of physics and calculus at the sophomore level, but in spite of this the author manages to cover all the important elements of dynamical meteorology. In addition there are descriptions of such developments as numerical weather prediction, both explaining the difficulties inherent in the subject and indicating the manner in which the problem has been tackled. Finally, a most important point, there are exercises of varying degrees of difficulty at the end of each chapter, as befits a book which will probably become the standard textbook in the subject for some time to come.

As the author remarks in the Preface, this book is intended primarily for those desiring an introduction to theoretical meteorology or those requiring a review of the subject, and the reviewer can recommend it to both classes of readers. *M. H. Rogers* (Bristol)

2457:

**Philip, J. R. The theory of local advection. I.** J. Meteorol. **16** (1959), 535–547.

A two-dimensional study is made of vertical diffusion in a horizontal convection field under steady state conditions. Both the velocity and diffusivity are allowed to have a power law dependence upon the height, $z$. The concentration is zero on the vertical surface, $x=0$, and either Dirichlet Neumann or Robin conditions are imposed on $z=0$, $x>0$. Solutions of micro-meteorological interest are constructed by similarity methods for the cases where constant flux or constant concentration is prescribed at $z=0$, $x>0$. These are extended by vigorous approximation to the radiation boundary condition case and a numerical analysis is given for a velocity profile of practical interest. If a steady vertical flux is imposed on the previous problems, an inversion surface can occur at which the vertical concentration gradient reverses. The position of such a surface is calculated for two cases.

*A. F. Pillow* (Toronto)

2458:

**Pone, R. Perspectives de la prévision numérique des précipatitions.** La Météorol. **1959**, 303–313. (English and Spanish summaries)

Author's summary: "During the last decade, the quantitative forecasting of the state of the atmosphere has made rapid progress due to the facilities offered by the electronic computers; there however remains much to be done to improve the results thus obtained and specially to achieve the forecasting of hydrometeors.

"As far as precipitation is concerned, the results so far obtained—by calculating the anticipated vertical motion —are not yet satisfactory.

"By taking into account the orographic effects and the fine structure of the atmosphere, an improvement can be expected for the short-range forecasts (24 to 48 hours).

"The fundamental obstacle that remains is the strong dispersion of the intensity of the precipitation for neighbouring places specially in the case of showers and storms. The forecast for a given place will therefore necessarily effect the parameters of the law of probability."

*J. F. Blackburn* (New York)

2459:

**Flinn, E. A. Local earthquake location with an electronic computer.** Bull. Seismol. Soc. America **50** (1960), 467–470.

Author's summary: "A straightforward least-squares iterative procedure for locating local earthquakes using only the direct waves $P_g$ and $S_g$ is now in use at the Australian National University. An IBM 650 electronic computer is used for all calculations, including estimates of the probable error of epicentral coordinates, depth of focus, and origin time."

2460:

**Jobert, G. Perturbations des marées terrestres.** Ann. Géophys. **16** (1960), 1–55. (English and Esperanto summaries)

Author's summary: "In the first part of this work, the deformations of the terrestrial crust, due to surficial loads, are examined. We determine the influence of an exponential variation of the elastic properties with depth and that of irregularities of the surficial topography.

"In a second part the thermoelastic deformations of the crust are studied. We demonstrate that only very localized variations of the surficial temperature can produce perturbations of the terrestrial tides."

2461:

**Ingram, R. E. An integral solution of the electromagnetic seismograph equation.** Bull. Seismol. Soc. America **50** (1960), 461–465.

Author's summary: "In investigating the response of an electromagnetic seismograph to various ground movements it is advantageous to have the solution of the differential equation as an integral. This is done by finding the Green's function, $f(s)$, for the particular instrument. The angular movement of the galvanometer is then

$$\theta(t) = q \int_0^t f(s)x''(t-s)ds,$$

where $x(t)$ is the ground movement and prime stands for the operator $d/dt$. It is sufficient to find one function, $F(s)$, with $dF/ds=f(s)$, to give the response to a displacement test, a tapping test, or a ground movement."

2462:

**Satô, Yasuo. Synthesis of dispersed surface waves by means of Fourier transform.** Bull. Seismol. Soc. America **50** (1960), 417–426.

2463:

**Stauder, William. $S$ waves and focal mechanisms: the state of the question.** Bull. Seismol. Soc. America **50** (1960), 333–346.

Author's summary: "The questions usually discussed with respect to the use of $S$ waves in determining the character of an earthquake focus concern the ambiguity of the fault-plane solution from $P$ waves and the problem of a single couple vs. a double couple as the model of an earthquake focus. Recent papers on the subject of earthquake mechanisms bring forward more basic questions concerning the adequacy of focal models. The theory of point sources and previous applications of $S$ waves to mechanism studies are reviewed as guides to further investigation."

2464:

**Savarenskiĭ, E. F. Determination of group and phase velocity from observations.** Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 1550–1559. (Russian)

An account of the theoretical basis of methods for the determination of group and phase velocities of observed seismic waves. *S. K. Zaremba* (Swansea)

2465:

**Mühlig, F. ★Grundlagen und Beobachtungs-Verfahren der astronomisch-geodätischen Ortsbestimmung.** Sammlung Wichmann, Bd. 20. Herbert Wichmann Verlag, Berlin, 1960. 131 pp. DM 26.00.

Despite its title, this book is not concerned with the determination of place, although the methods described are in principle identical with those of celestial navigation. They amount to the precise determination of the space direction (right ascension and declination) of the zenith of the observer at a known Greenwich time. Positions so derived are referred to by geodesists as astronomic positions; they are unsuitable for precise surveys because they are affected in an irregular way by attracting masses within the earth. For the same reason, however, they are of interest to the geophysicist, when compared with positions deduced from surveys on the basis of an exact ellipsoid of revolution: the discrepancies, called the deflections of the vertical, tell something about the subterranean distribution of density. This book is concerned with the precise measurement of astronomic position, and with astronomic measurements of azimuth for the control of triangulation.

The first half of the book covers the theory of the angular motion of the earth with respect to the stars, with several valuable digressions including the theory of the variation of latitude, a modern theory of refraction, and a reference to the discrepancy between the inertial frame as determined from the stars and as determined from motions in the solar system, which, it is suggested, is of fundamental importance.

No reference is made to the paradoxes of azimuth determination, or to azimuth determination in polar areas. Nor is there any discussion of lunar or satellite methods of position determination.

The bibliography is very brief.

*J. A. O'Keefe* (Chevy Chase, Md.)

2466:

**Baeschlin, C. F. Rapport spécial sur le nivellement et la pesanteur.** Bull. Géodésique (N.S.) No. 57 (1960), 245–298.

2467:

**Hristow, W. K. Klassische und moderne Genauigkeitsabschätzungen in einigen Ausgleichungsfällen.** Acta Tech. Acad. Sci. Hungar. **23** (1959), 17–43 [Geodesy Congress, Sept. 18–21, 1956].

Die Begriffe der Statistik werden für die Ausgleichungsrechnung nach der Methode der kleinsten Quadrate nutzbar gemacht. Insbesondere sind es der Korrelationskoeffizient, die Chi-Quadrat- und die $t$-Verteilung, der Begriff der Orthogonalität und des Vertrauensbereiches, die eingehend diskutiert werden. Als Beispiel wird die Bildung des arithmetischen Mittels behandelt. Diskussionsbemerkungen von Böhm, Arnold, Reichenecker, Peschel, Durnjew schließen sich an. *H. Wolf* (Bonn)

2468:

**Ansermet, A. Le fractionnement des calculs de compensation et la détermination des poids en cas de covariance.** Schweiz. Z. Vermessg. Kulturech. Photogr. **58** (1960), 231–239.

2469:

**Baeschlin, C. F. Das Geopotential, metrische Höhen und Gebrauchshöhen. Eine Neuerung auf dem Gebiet der höheren Geodäsie.** Schweiz. Z. Vermessg. Kulturtech. Photogr. **58** (1960), 191–205.

2470:

**Frisch, Armand K. Die Methode der direkten Geländekorrektion. I.** Schweiz. Z. Vermessg. Kulturtech. Photogr. **58** (1960), 240–253.

2471:

**Frisch, Armand K. Die Methode der direkten Geländekorrektion. II.** Schweiz. Z. Vermessg. Kulturtech. Photogr. **58** (1960), 271–285.

2472:

**Lo, So-fen. The problem of measuring land surface area on topographical maps.** Acta Geodet. Cartograph. Sinica **4** (1960), 11–18. (Chinese. English summary)

Author's summary: "The author attempts to prove by the theory of integral calculus that the land surface area obtained by the method of N. M. Volkov will always be smaller than the actual area. Furthermore, the author proves by concrete examples that N. M. Volkov's method is applicable only in measuring the surface land of a conic body. According to the horizontal distance and height of contour intervals on topographical maps, the author makes a slope ruler for measuring slope angles in all cases of surface relief. With it, one could delimit different slope regions the area of which could be thus calculated. This method, as the author holds, possesses more theoretical reliability as well as facility of actual work of measurement."

2473:

**Tang, Sung-shih. Some opinions about "Adjustment of observations by method of integration". I.** Acta Geodet. Cartograph. Sinica **3** (1959), 247–252. (Chinese. English summary)

Author's summary: "The author points out certain errors in C. Tung's essay [same Acta **1** (1957), 171–180]."

2474:

**Yeh, Sho-An. Some opinions about "Adjustment of observations by method of integration". II.** Acta Geodet. Cartograph. Sinica **3** (1959), 253–254. (Chinese)

Author's summary: "The author points out certain errors in C. Tung's essay [same Acta **1** (1957), 171–180]."

2475:

**Homoródi, L. Die Deutung des Punktfehlers.** Acta Tech. Acad. Sci. Hungar. **23** (1959), 45–57 [Geodesy Congress, Sept. 18–21, 1956].

Es wird vorgeschlagen, anstelle des mittleren Punktfehlers $M$ Helmertscher Definition einen "durchschnittlichen" mittleren Punktfehler $K$ einzuführen, wobei $K\sqrt{2}=M$ ist. Diskussionsbeiträge von Reissmann, Vincze und Durnjew ergänzen die Darlegungen.

*H. Wolf* (Bonn)

2476:

**Tárczy-Hornoch, A. Weiteres zur Ausgleichung der kontinentalen Triangulierungsnetze.** Acta Tech. Acad. Sci. Hungar. **23** (1959), 9–16 [Geodesy Congress, Sept. 18–21, 1956].

Zur Ausgleichung eines Triangulationsnetzes nach der Methode der kleinsten Quadrate wird ein fingiertes übergeordnetes Netz gebildet, dessen Winkel aus dem tatsächlich beobachteten Netz abgeleitet werden. Diese Winkel werden als fingierte Beobachtungen einer weiteren Ausgleichung unterzogen. Der Verf. zeigt am Beispiel eines sechsstrahligen Zentralsystems die Ausgleichung nach vermittelnden Beobachtungen mit Bedingungsgleichungen unter Benutzung von Helmerts Theorie der äquivalenten Beobachtungen und gibt dieser Ausgleichungsform den Vorrang vor den bedingten Beobachtungen, wenn es sich um eine gemeinschaftliche Ausgleichung von Winkeln und Seiten in einem Dreiecksnetz handelt. *H. Wolf* (Bonn)

2477:

**Kašpar, J. Das Bild der Kurve bei der Abbildung einer beliebigen Fläche auf eine andere.** Acta Tech. Acad. Sci. Hungar. **23** (1959), 131–146 [Geodesy Congress, Sept. 18–21, 1956].

Bei dem allgemeinen Problem der Abbildung einer beliebigen Fläche auf einer anderen wird nach Maßgabe der von Laborde für einen Sonderfall durchgeführten Ableitung der Richtungs- und Streckenreduktion gefunden, daß das Gaußsche Krümmungsmaß bereits im Glied 3. Ordnung und seine Veränderung im Glied 4. Ordnung der neu ermittelten Formel für die Richtungsreduktion enthalten ist. Abschließend werden Formeln für die rechtwinklig-geodätischen Koordinaten des Bildpunktes angegeben. *H. Wolf* (Bonn)

2478:

**Hazay, I. Über die Probleme der Projektion zwischen zwei Ellipsoiden.** Acta Tech. Acad. Sci. Hungar. **23** (1959), 121–129 [Geodesy Congress, Sept. 18–21, 1956].

Über die verschiedenen Möglichkeiten zur konformen Abbildung eines Ellipsoides auf einem anderen Ellipsoid wird kurz referiert: (1) mit längentreuem Normalparallel und gleicher Normalbreite; (2) mit längentreuem Normalparallel und geringstmöglicher Änderung des Bildmaßstabes; (3) mit gleicher Normalbreite und gleichen Längen; (4) mit vorgegebener Änderung der Normalbreite. Darüberhinaus werden noch verschiedene Verfahren zur Neu-Orientierung des Dreiecksnetzes auf dem Bildellipsoid angegeben. Hieran schließen sich Diskussionsbeiträge von Böhm, Arnold und Hristow. *H. Wolf* (Bonn)

2479:

**Szádeczky-Kardoss, Gy. Die Berechnung der ellipsoidischen Bogenlänge von Meridianschnitten und Normalschnitten mittels Rechenmaschine.** Acta Tech. Acad. Sci. Hungar. **23** (1959), 147–152 [Geodesy Congress, Sept. 18–21, 1956].

Die Berechnung der Meridianbogenlänge wird auf der Grundlage einer Rekursionsformel durchgeführt und in eine Reihe entwickelt, die nach den ungeraden Potenzen des cos der geogr. Breite fortschreitet. Anschließend werden Näherungsformeln für kürzere Bögen aufgestellt, eine Anwendung gelegentlich der Bogenlängenbestimmung für einen Normalschnitt gezeigt, und ein Zahlenbeispiel wird vorgeführt. *H. Wolf* (Bonn)

## OPERATIONS RESEARCH, ECONOMETRICS, GAMES

See also A1726, 1988, 2454.

2480:

**Azorín Poch, Francisco. Some statistical problems in the construction of consumption scales.** Trabajos Estadíst. **10** (1959), 63–73. (Spanish. English summary)

2481:

**Morishima, Michio. Some properties of a dynamic Leontief system with a spectrum of techniques.** Econometrica **27** (1959), 626–637.

The author describes a generalization of the Leontief stock-flow model to admit alternative activities for producing each commodity (but without joint production). Under standard assumptions, his main result is that there exists a choice of activities, one for each commodity, with the property that its associated price-vector (labor is "numeraire") is component by component less than or equal to that for any other choice. For one plausible price-adjustment process, the sequence of price vectors is shown to converge to this special one from arbitrary initial position. There are some results on the associated path of outputs. *R. Solow* (Cambridge, Mass.)

2482:

**Wurtele, Zivia S. A note on some stability properties of Leontief's dynamic models.** Econometrica **27** (1959), 672–675.

The author notes that the usual instantaneous formulation of the Leontief model $X(t)=AX(t)+BX'(t)+Y(t)$, where $X$ and $Y$ are column vectors and $A$ and $B$ are square matrices of compatible dimension and with the usual properties, has been criticized because even with $Y$ constant, $X(t)$ is unbounded. She shows that the backward-difference formulation

$$X(t) = AX(t)+B([X(t)-X(t-\tau)]/\tau)+Y(t),$$

for $\tau$ sufficiently large, is always stable in the sense that with $Y$ constant, $X(t)\to(I-A)^{-1}Y$. She does not comment on the fact that to achieve stability in this way it is required that all the real characteristic numbers be negative. The one-dimensional numerical example (incorrectly given) exhibits this clearly.

*R. Solow* (Cambridge, Mass.)

2483:

**Nikaido, Hukukane. Recent topics in mathematical economics.** Sûgaku 8 (1956/57), 40–53. (Japanese)

A neat survey (with brief but excellent economic exposition) of central theorems on the existence of economically meaningful solutions in some important economic models. § 1 covers the existence problem of the social welfare function discussed by K. J. Arrow [*Social choice and individual values*, Wiley, New York, 1951; MR **12**, 624] and K. Inada [Ann. Inst. Statist. Math. Tokyo **6** (1954), 115–122; MR **16**, 386]. § 2 is an introductory summary of various forms of the fixed point theorem by S. Kakutani, Eilenberg and Montgomery, Begle, Fan, Glicksberg, Arrow and Debreu, Nikaido, Schauder, and Tychonoff. § 3 concisely presents the applications of these theorems to economic models for which the Walras' Law holds and to a non-cooperative game. They cover the problems discussed by Arrow and Debreu, Nikaido, and McKenzie. *S. Ichimura* (Osaka)

2484:

**Frisch, Ragnar. A complete scheme for computing all direct and cross demand elasticities in a model with many sectors.** Econometrica **27** (1959), 177–196.

This paper starts with a restatement of the classical theory of consumer's choice emphasizing the "money flexibility", the elasticity of the marginal utility of income with respect to income itself. The effect of a change in price is analyzed for the case where the marginal utility of money is held constant by a compensating change in income. Then "want-independence" is defined by the vanishing of the second-order cross-derivatives of the utility function (with due recognition that this concept implies cardinal utility). It is shown that under want-independence the number of elasticities to be estimated is drastically reduced; all own- and cross-price elasticities can be derived from a knowledge of the income elasticities and one own-price elasticity (or the money flexibility). From empirical work due to L. Johansen (unpublished) it appears that for three commodity groups the income and price elasticities exhibit the relation corresponding to want-independence, with a money flexibility of about minus two. An extension of the theory to grouped commodities is outlined. In an appendix integrability conditions are discussed, with special reference to the transition from local to global choice.

*H. S. Houthakker* (Cambridge, Mass.)

2485:

**Robinson, Enders A. A stochastic diffusion theory of price.** Econometrica **27** (1959), 679–684.

On the assumption that $x_t = \bar{x}_t(p_t) + u_t$; $p_{t+\tau} = p_t + \pi$ (if $x_t \geq 0$), $p_{t+\tau} = p_t - \tau$ (if $x_t < 0$); $\bar{x}(p) = -ap$ $(a > 0)$; and the variates $u_t$ have identical rectangular distributions with range $-aR \leq u_t \leq aR$, where $\bar{x}(p)$ is the excess demand function of the price, $p$ the deviation from its equilibrium price and $\tau$ and $\pi$ are time and price units, the paper derives the one-step transition probability that $q_{ij} = \Pr\{p_{t+\tau} = j\pi \mid p_t = i\pi\}$ are

$$q_{i,i+1} = \Pr\{x_t \geq 0 \mid p_t = i\pi\} = \frac{1}{2} - \frac{i}{2R},$$

$$q_{i,i-1} = \Pr\{x_t < 0 \mid p_t = i\pi\} = \frac{1}{2} + \frac{i}{2R},$$

$q_{ij} = 0$ for $j \neq i+1$, $i-1$. This is the random walk of the Ehrenfest model of diffusion. By reducing the Kolmogorov equation for Markov processes [A. Kolmogorov, Math. Ann. **104** (1931), 415–458] to a difference equation the paper proves that $E\{p_t \mid p_0\} = p_0 (1-(1/R))^t$; that is, $p_t$ damps out exponentially to the equilibrium price.

*S. Ichimura* (Osaka)

2486:

**Maffei, Richard B. Brand preferences and simple Markov processes.** Operations Res. 8 (1960), 210–218.

Author's summary: "Brand preference information combined with a simple two-dimensional Markov process can be used to study characteristics of market dynamics. Advertising activity can be used to alter temporarily the brand preference structure of the consuming public. Such changes generate interesting transient conditions that can be studied mathematically. Discussions of time relations, period-to-period changes in market shares, gains and losses resulting from promotional activity and rapidity of convergence to new steady-state values are all considered. In a concluding section, sensitivity characteristics of the relations are commented upon."

2487:

**McGuinness, John S. A managerial game for an insurance company.** Operations Res. 8 (1960), 196–209.

2488:

**Heller, J. Some numerical experiments for an $M \times J$ flow shop and its decision-theoretical aspects.** Operations Res. 8 (1960), 178–184.

Author's summary: "Numerical experiments for $M \times J$ deterministic flow shops performed on an IBM 704 digital computer lead to the conclusion that the schedule times are approximately normally distributed for large numbers of jobs. The meaning of this result in the decision-theoretical problem of sampling for minimum is discussed. Examples of these results for $10 \times 100$ and $10 \times 20$ schedules are given."

2489:

**Debeau, David; James, George; Drozda, William. Component replacement liability.** Management. Sci. **6** (1960), 295–302.

Authors' summary: "This paper reports the results of an analytic problem arising out of a history. Although some of the context and all of the numbers have been changed to protect proprietary information, the following important facet of the original case history remains invariant in the transformation. Management of the sponsoring firm had planned to make a policy decision based on differences between averages of overlapping distributions. After the influence of the probability distributions was evaluated and explained, management reversed its intended policy decision."

2490:

**Mayer, Raymond R. Problems in the application of replacement theory.** Management. Sci. **6** (1960), 303–310.

Author's summary: "A great deal of time and effort has been devoted to the development of quantitative methods

for the solution of business management problems. Unfortunately, a tremendous difference exists between the rate at which these methods are being generated and the rate at which they are being applied. This suggests that there exist problems of application which industry is not able to overcome. The purpose of this paper is to present and analyze the difficulties encountered in a single, but representative, area of decision-making, namely, equipment replacement. It is hoped a presentation and analysis of these difficulties will have some effect on the nature of future research activities."

2491:

**Nordbotten, Svein. Linear programming and automatic computers.** Skand. Aktuarietidskr. **1959**, 61–72.

Survey article.

2492:

**Dantzig, George B. On the status of multistage linear programming problems.** Management Sci. **6** (1959/60), 53–72.

Author's summary: "A survey paper which discusses by means of examples how a dynamic linear programming problem of the transportation type may be reduced to a sequence of single period problems in the manner of dynamic programming."

*M. J. Beckmann* (Providence, R.I.)

2493:

**Roy, Bernard. Contribution de la théorie des graphes à l'étude de certains problèmes linéaires.** C. R. Acad. Sci. Paris **248** (1959), 2437–2439.

In a graph with nodes $X$ a real-valued function $a(x, y)$ is defined on certain edges $(x, y)$. Theorem 1: There exists a function $t$ on $X$ such that $t(y)-t(x)\geq a(x, y)$ if and only if the sum of $a(x, y)$ around any circuit is non-positive. The remainder of the note is concerned with finding such functions $t$ which minimize $\sum_{x\in X} t(x)c(x)$ when $c$ is a preassigned function on $X$. This is nothing more than the dual of the usual transportation problem with the signs changed, and theorems 2 and 3 essentially show that this problem can be solved by the methods of Kuhn, and Ford and Fulkerson.

*D. Gale* (Providence, R.I.)

2494:

**Fulkerson, D. R. A network-flow feasibility theorem and combinatorial applications.** Canad. J. Math. **11** (1959), 440–451.

Let $G$ be be a finite directed network consisting of a set $N$ of nodes $x, y, \cdots$ and directed arcs joining pairs of nodes. The arc from $x$ to $y$ is denoted by $(x, y)$. Let there be associated with each arc $(x, y)$ a "capacity" $c(x, y)$. Here $c(x, y)$ is a non-negative real or plus infinity. The set $N$ of nodes is partitioned into three subsets. These subsets consist of the set $S$ of "sources", the set $T$ of "sinks", and the set $R$ of "intermediate nodes". Let $A(x)$ denote the set of nodes $y$ such that $(x, y)$ is an arc, and let $B(x)$ denote the set of nodes $y$ such that $(y, x)$ is an arc. A real-valued function $f$ defined on the arcs of $G$ is called a flow from $S$ to $T$ provided

$$(1)\qquad \sum_{y\in A(x)} f(x, y) = \sum_{y\in B(x)} f(y, x) \quad (x\in R),$$

$$(2)\qquad 0 \leq f(x, y) \leq c(x, y) \quad (\text{all } (x, y)).$$

We consider now flows from $S$ to $T$ that satisfy bounds on the net flow leaving each $x\in S$ and entering each $x\in T$. For each $x\in S$ let $\alpha(x)$ and $\beta(x)$ satisfy $0\leq\alpha(x)\leq\beta(x)$ and for each $x\in T$ let $a(x)$ and $b(x)$ satisfy $0\leq a(x)\leq b(x)$. The further constraints

$$(3a)\quad \alpha(x) \leq \sum_{y\in A(x)} f(x, y) - \sum_{y\in B(x)} f(y, x) \leq \beta(x) \quad (x\in S),$$

$$(3b)\quad a(x) \leq \sum_{y\in B(x)} f(y, x) - \sum_{y\in A(x)} f(x, y) \leq b(x) \quad (x\in T),$$

are called feasible provided there exists a flow from $S$ to $T$ satisfying them. We are now in a position to state the main feasibility theorem derived in this paper. The constraints (3a) and (3b) are jointly feasible if and only if the constraints

$$\alpha(x) \leq \sum_{y\in A(x)} f(x, y) - \sum_{y\in B(x)} f(y, x) \quad (x\in S),$$

$$\sum_{y\in B(x)} f(y, x) - \sum_{y\in A(x)} f(x, y) \leq b(x) \quad (x\in T),$$

and

$$\sum_{y\in A(x)} f(x, y) - \sum_{y\in B(x)} f(y, x) \leq \beta(x) \quad (x\in S),$$

$$a(x) \leq \sum_{y\in B(x)} f(y, x) - \sum_{y\in A(x)} f(x, y) \quad (x\in T),$$

are separately feasible. The feasibility theorem is not established directly but rather by way of the max-flow min-cut theorem of Ford and Fulkerson.

The feasibility theorem has a number of combinatorial applications. In section 5 the feasibility theorem is used to generalize the existence theorem of Gale and Ryser for (0, 1)-matrices of prescribed row and column sums. Section 6 generalizes Ore's theorem on the necessary and sufficient conditions that a finite directed graph $G$ have a subgraph $H$ possessing specified local degrees. The concluding application in section 7 yields the theorem of Hoffman and Kuhn arising in the theory of systems of distinct representatives.

*H. J. Ryser* (Columbus, Ohio)

2495:

**Berman, Edward B. A regional production and transportation model.** Management Sci. **5** (1959), 319–326.

A linear programming model is set forth for maximizing a vector of final demand deliveries under both regional production constraints and transportation network constraints. The model is essentially designed to study short-run effects, and hence no provision is made for the expansion of either industrial or transportation capacities. Within available capacities, however, the location of production is chosen so as to maximize final demand deliveries. A rough dynamic version of the model is presented as well as an algorithm for reaching an approximation of the optimum solution and thereby reducing the computational burden.

*S. Ichimura* (Osaka)

2496:

**Karlin, Samuel. ★Mathematical methods and theory in games, programming and economics. Vol. I: Matrix games, programming, and mathematical economics. Vol. II: The theory of infinite games.** Addison-Wesley Publishing Co., Inc., Reading, Mass.-London, 1959. Vol. I, x+433 pp. $12.50. Vol. II, xi+386 pp. $12.50.

"We may summarize our aims as an attempt to unify

the mathematical techniques of the field and to help crystallize the concepts underlying these kinds of decision problems. It is not our purpose to propose a formal structure which will encompass all the problems in the areas covered." Vol. I treats static decision situations where the strategy spaces are finite, part I being devoted to two-person zero-sum matrix games and part II, to programming and topics in mathematical economics. Vol. II, which is part III, explores static situations where the strategy spaces are infinite. "The coverage of these volumes is in no sense exhaustive; I have stressed most what appeals to me." The last statement applies especially to vol. II, which seems more detailed than warranted given that much of the material is readily available in the Annals of Mathematics Studies 24, 28, and 39.

These are mathematical books on branches of mathematics that have either arisen out of or are applied to social science problems; they are not, however, concerned with this parentage as such. To be sure, some motivation is given for each model, but generally they are stated without much ado and the main concern is with detailed, careful analyses of their properties. Considerable care is taken to discuss and to motivate each analysis so that the nature and crux of the mathematical argument is not obscured by the details. Both books are designed as texts at the advanced undergraduate and at the graduate levels, as well as for reference, and they are self-contained in the sense that concise (52 pp.) appendices are included on vector spaces and matrices, convex sets and functions, semi-continuous and continuous functions, fixed point theorems, and set functions. They are in reality only self-contained for the student who has had considerable mathematical experience or who has detailed assistance.

A short introduction that delineates the major outlines of the field begins vol. I, which then turns to part I on matrix games. The min-max theorem is proved in three ways: by a separation theorem for convex sets, by the Kakutani fixed-point theorem, and by properties of convex functions. Properties of the optimal strategy sets are then developed, a fundamental relation on the dimensions of these sets is established, games are constructed having prescribed sets of optimal strategies, and those games with unique solutions are shown to constitute an open dense subset of all games of a given order. Using some of these results, solutions are found (ch. 4) to five specific games: a Colonel Blotto problem, identification of friend and foe, a simplified version of poker, an election campaign model, and a union-management bargaining model.

Part II begins with linear programming, presenting the usual fundamental existence and duality theorems, the relations to game theory, and a variety of famous illustrations including the warehouse, optimal assignment, and transportation problems. Ch. 6 is devoted to computational methods—primarily the simplex algorithm and modifications of it—for finding solutions to linear programs and games and to techniques for determining the value of a game. The topic of the next chapter is nonlinear programming, primarily concave and convex programming. A game interpretation is given, the Arrow-Hurwitz gradient method of solution is developed, and Fenchel's theory of conjugate functions is presented and used to prove a duality theorem. The final two chapters of this volume treat portions of mathematical economics where the mathematical methods are the same or similar to those used for games and programming. Included are the linear Leontief model of exchange and a non-linear production generalization; a theory of consumer choice; three non-linear models of competitive equilibrium whose existence theorems rest upon some version of the Kakutani fixed-point theorem; relations between Pareto optimality and the competitive equilibrium; the stability of the equilibrium; and the von Neumann model of an expanding economy and generalizations of it.

Part III, i.e., vol. II, starts with a reprint of ch. 1 of vol. I. Ch. 2 introduces the major issues, classifications, and tacks taken with infinite games. The remaining seven chapters examine a variety of more or less special infinite games: separable and polynomial games, games of timing, those with analytic and bell-shaped kernels, several duels, and a series of variants of poker. The central problems in each case are to establish the min-max theorem, or a weakened version of it, and to devise means for finding optimal strategies. These theories, many of which are due to the author and his ex-colleagues at RAND, are well known to be subtle and complex. The volume ends with a reprint of the appendices and the 257 item bibliography of vol. I. This makes the two volumes independent, but for a person owning vol. I, it only increases the already high marginal cost of getting vol. II.

Each chapter ends with a relatively long series of problems of varied difficulty, some of which extend the theory, and with a set of detailed, useful historical notes. Answers or hints to most of the problems are included at the end of each volume.

Some of the results and a number of the proofs are new.

With these volumes, we now have a rigorous and reasonably readable advanced text in the several aspects of static, non-statistical decision theory.

*R. D. Luce* (Philadelphia, Pa.)

## BIOLOGY AND SOCIOLOGY

See also A1589, A1726, 2037, 2038.

2497:

**Krakau, C. E. T. A note on the Fourier transform of Lorente de Nó's potential function of the external field of a nerve in a volume conductor.** Kungl. Fysiogr. Sällsk. i Lund Förh. **27** (1957), 177–183.

2498:

**Le Roy, Henri Louis. ★Statistische Methoden der Populationsgenetik: Ein Grundriss für Genetiker, Agronomen und Biomathematiker.** Reihe der experimentellen Biologie, Bd. 15. Lehrbücher und Monographien aus dem Gebiete der exakten Wissenschaften. Birkhäuser Verlag, Basel und Stuttgart, 1960. 397 pp. Fr. 67.50.

This book is devoted to the mathematical theory of the inheritance of continuously varying characters controlled by many genes. In the first chapter the author discusses the analysis of variance of such characters into linear genetic, dominance, epistatic and environmental components. The second chapter deals with path coefficients and the correlation of relatives, the third with the estimation of components of variation and the fourth and largest gives an extensive discussion of selection methods and the theory underlying them. The mathematics is at

a level suitable to a statistically minded biologist and is given in great detail with many graphs and numerical examples. There is a good bibliography and the book is a useful text for the applied mathematical geneticist.

*P. A. P. Moran* (Oxford)

2499:

**Bennett, B. M. Note on the power function of the $X_s$ test in genetics.** Skand. Aktuarietidskr. **1959**, 1–5.

2500:

**Bennett, Joseph F.; Hays, William L. Mutidimensional unfolding: determining the dimensionality of ranked preference data.** Psychometrika **25** (1960), 27–43.

Authors' summary: "A model is proposed which treats rankings given by a group of judges as representing regions in an isotonic space of dimensionality $r$. Three possible criteria for estimating lower bound dimensionality are discussed: mutual boundary, cardinality, and the occurrence of transposition groups. Problems associated with each criterion are mentioned."

2501:

**Gibson, W. A. Remarks on Tucker's inter-battery method of factor analysis.** Psychometrika **25** (1960), 19–25.

## INFORMATION AND COMMUNICATION THEORY

See also A1780, 2035.

2502:

**Stumpers, F. Louis H. M. A bibliography of information theory (Communication theory-cybernetics). (Third supplement).** Trans. I. R. E. **IT-6** (1960), 25–51.

2503:

**Haussig, D. Zusammenstellung und Vergleich bekannter Verfahren zur Auffindung periodischer Pulse bei Anwesenheit von Geräuschen.** Hochfrequenztech. Electroak. **69** (1960), 94–103.

2504:

**Boisvert, Maurice. Les diagrammes de fluence de signal.** Ann. Télécommun. **13** (1958), 50–77.

2505:

**Blair, Charles R. A program for correcting spelling errors.** Information and Control **3** (1960), 60–67.

The author investigates a number of techniques for obtaining a machine program that will recognize misspelled words. The technique favored here requires that the machine be provided with a list of correctly spelled words in its memory. To each such word and to each word read by the machine is assigned an abbreviation as follows. Each letter of a word is given a score which depends on its name and on its position in the word. The four letters having the highest scores comprise the abbreviation sought. If a word read is different from a memory word but has the same abbreviation, it is considered to be a misspelling of that word. In one example a machine identified correctly 89 of 117 misspelled words, and 3 of these required longer abbreviations. Two words were incorrectly identified. Two types of deficiency are noted and corrections for these types of errors are discussed.

*E. J. Cogan* (Bronxville, N.Y.)

## SERVOMECHANISMS AND CONTROL

2506:

**Solodovnikov, V. V. ★Introduction to the statistical dynamics of automatic control systems.** Translation edited by John B. Thomas and Lotfi A. Zadeh. Dover Publications, Inc., New York, 1960. xx+307 pp. Paperbound: $2.25.

[The first Russian edition was reviewed in MR **14**, 388.]

This well-edited paperbound book is concerned with statistical aspects of the theory of linear control systems in the sense of Wiener and Kolmogorov; see, e.g., Y. W. Lee's recent book, *Statistical theory of communication* [John Wiley, New York, 1960]. The book under review does not consider contributions to the theory of filtering or prediction that have been made since 1952. Indeed, there is no consideration of nonlinear problems in the sense of Wiener [*Nonlinear problems in random theory*, Wiley, New York, 1958; MR **20** #7337].

In particular, chapter 1 deals with the frequency domain characteristics of linear dynamical systems, i.e., generalized harmonic analysis. Chapter 2 consists of a heuristic treatment of those aspects of probability theory which are relevant to a study of random signals. Chapter 3 treats of stationary stochastic processes. Chapter 4 involves an analysis of linear systems subjected to probabilistically specified inputs. Chapter 5 considers methods of approximation of spectral density functions by means of meromorphic functions. Chapter 6 treats of least mean-squared error and the optimum transfer function for a linear system. Chapter 7 deals with optimum prediction, smoothing and differentiation in linear systems. The last chapter gives a treatment of optimum filtering and prediction for the case of a finite duration of observation. The text of the book terminates with a sequence of tables useful in the process of solving specific engineering problems.

The book is well-documented with references to the Russian literature on the stochastic nature of linear control systems.

*A. A. Mullin* (Urbana, Ill.)

2507:

**Balchen, Jens G.; Berre, Arne G. A method for evaluating the accuracy in the time domain associated with approximation in the frequency domain.** Acta Polytech. Scandinav. No. 262 (1959), 10 pp.

Authors' summary: "A method is outlined which makes it possible by simple means to estimate the error in the time response arising from a certain inaccuracy between an exact and an approximate system in the frequency domain. A new error criterion which makes use of logarithmic amplitude and frequency plots is introduced."

*J. Hartmanis* (Schenectady, N.Y.)

2508:

**Krasovskiĭ, N. N. On a problem of optimum control of nonlinear systems.** Prikl. Mat. Meh. **23** (1959), 209–229 (Russian); translated as J. Appl. Math. Mech. **23**, 303–332.

Consider a vector differential equation (1) $dx/dt = f(x, t)+q(t)\eta(t)$, where $f(x, t)$ and $q(t)$ are known vector functions. Further assume that $f(x, t)$ is continuously differentiable for $t \geqq t_0$ at all points of the space $x$, except at the points lying on the surfaces (2) $\xi_\alpha(x, t)=0$ $(\alpha=1, \cdots, \mu)$ of the space $x \times t$. The function $q(t)$ is assumed to be piecewise smooth and to have only a finite number of discontinuities of the first kind in every bounded interval $t_0 \leqq t \leqq T$. The surfaces (2) do not intersect by assumption, and the functions $\xi_\alpha$ are continuously differentiable. It is required to determine a piecewise smooth control function $\eta=\eta^0(t)$ $[|\eta(t)| \leqq 1]$ such that, given an initial instant $t=t_0$, a point $x=x_0$ and a smooth curve $x=z(t)$, the moving point $x(x_0, t_0, t, \eta^0)$ of the trajectory of (1), where $\eta(t)=\eta^0(t)$, reaches the curve $x=z(t)$ in the shortest possible time $T^0=t-t_0$ [see also Boltyanskiĭ, Gamkrelidze and Pontryagin, Dokl. Akad. Nauk SSSR **110** (1956), 7–10; MR **18**, 859; Gamkrelidze, ibid. **116** (1957), 9–11; MR **20** #1016; Boltyanskiĭ, ibid. **119** (1958), 1070–1073; Krasovskiĭ, Avtomat. i Telemeh. **18** (1957), 960–970; MR **20** #803].

In the present paper certain existence problems as well as necessary and sufficient criteria for the existence of optimum trajectories of (1) with discontinuities (2) are considered. The argument is carried out for the general case of $n$-dimensional vector functions mentioned above. However, due to certain circumstances, an efficient formulation of the theorems is difficult for $n>2$. The author takes particular care to point out all cases for which the assertions are correct only when $n=2$. An example is given to illustrate the conditions imposed on equation (1) in proving the theorems of the paper. Due to the complexity and a rather large number of definitions, lemmas, theorems and corollaries involved the reader is referred for more detailed results to the paper itself.

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2509:

**Morris, D. J.; Alexander, W. An introduction to the ternary code number system.** Electronic Engrg. **32** (1960), 554–557.

Authors' summary: "This article reviews the information concerning ternary arithmetic, gives a representation of the ternary system and its conversion to and from the binary and decimal numerical systems."

# AUTHOR INDEX

## PART B